APPENDIX A

NOMENCLATURE

Underwater acoustics is a multi-discipline field involving concepts from fluid mechanics, aerodynamics, thermodynamics, electrical, mechanical and marine engineering and naval architecture, as well as acoustics. When possible, the author has selected symbols and abbreviations familiar to workers in each of these fields. In cases where this would cause confusion due to duplication preference has usually been given to acoustics.

Over 450 symbols and abbreviations used in this volume are listed here. Common symbols and abbreviations used in more than one chapter are given first. Nomenclature restricted to only one or two chapters is then listed by chapter. In most instances reference is made to the page, equation or figure where the symbol is first used and/or defined. A few symbols that occur only once in the text and are defined where they occur have been omitted.

A.1 General Nomenclature

Roman Letters

$\vec{A}, \vec{B}$	general vectors	p. 11
A, B, C	amplitudes; coefficients; constants	
$\underline{A}, \underline{B}, \underline{C}, \underline{D}$	complex numbers; amplitudes; coefficients	
A_i, B_i	vector components	p. 11
$\underline{A}^*, \underline{B}^*$	complex conjugates	Eq. 1.52
A_r	radial component of $\vec{A}$	p. 14
a, b	coefficients; constants	
$\underline{a}, \underline{b}, \underline{c}, \underline{d}$	complex coefficients	
a	radius	
a_o	radius of radiating body	p. 57
B	bulk modulus	Eq. 2.53
b	width or thickness	
c	speed of wave propagation	Eq. 2.1
c_o	speed of sound in a fluid	Eq. 2.53
c_ℓ	longitudinal wave speed in a solid	Eq. 5.1
c_s	shear wave speed in a solid	Eq. 5.2
$D\_$	substantial; material derivative	p. 24
D	diameter	
D_o	dipole strength	p. 77
$d\_$	total derivative	
d	distance; separation	

Roman Letters (continued)

E	energy	
E_{ac}	acoustic (radiated) energy	Eq. 4.49
e	exponential	
F, f, G	functions	
$\vec{F}$	force	
$\underline{F}$	complex value of force	
F_o	force amplitude	
f	frequency	
Δf	bandwidth	
f_i	component of force vector per unit volume	p. 48
f_o	resonance frequency	p. 63
G	shear modulus	Eq. 5.2
$g, \vec{g}$	acceleration of gravity	Eq. 2.36
H, h	depth; thickness	
I	acoustic intensity	Eq. 2.69
i	$\sqrt{-1}$	Eq. 1.54
i, j, k	index numbers	
$\hat{i}, \hat{j}, \hat{k}$	unit cartesian vectors	Eq. 1.24
J_o, J_1	Bessel functions	
j	integer	
K	various parameters, coefficients	
k	wave number	Eq. 2.8
$\vec{k}$	wave vector	Eq. 2.13
k_i	components of wave vector	Eq. 2.13
L	length	
L_N, L_n	noise level (dB or dBs)	p. 10
L_S, L_s	source level (dB or dBs)	p. 7
ℓ	turbulence scale length (eddy size)	p. 54
M	Mach number	Eq. 1.2
M'	acoustic Mach number	Eq. 2.67
m, n	index numbers; integers; exponents	
m	mass	
m_e	entrained mass	Eq. 4.13
N	number of	
$\hat{n}$	unit vector normal to a surface	Eq. 1.47
n_i	direction cosines	Eq. 2.2
P	pressure amplitude	Eq. 2.75
P_A	static pressure in atmospheres	Eq. 4.40
p	pressure (instantaneous or rms)	
p'	acoustic pressure	Eq. 2.19
p_o	ambient static pressure; local steady value of instantaneous pressure	Eq. 2.20
$\underline{p}_o'$	monopole pressure field	Eq. 4.58

Roman Letters (continued)

Q	source stability; resonance sharpness	Eq. 1.18 and Eq. 5.138
Q	mass flux of a source	Eq. 3.27
Q_o	amplitude of mass flux	Eq. 4.1
q	mass flux per unit volume	Eq. 3.6
R	radius	
R	resistance	
R_i	input resistance	Eq. 5.116
R_r	radiation resistance	Eq. 3.1
r	radial coordinate; distance	
$\hat{r}$	radial unit vector	p. 14
S	area (cross-section or surface)	
S_o	mean area of radiating body	
s	distance along a path	
T	period of time	
t	time	
t'	retarded time	Eq. 2.6
U	fluid speed	
U_o	flow speed	p. 54
u	instantaneous surface velocity; fluid speed	Eq. 4.2
u_i	component of fluid speed	p. 53
u_o	amplitude of surface velocity	Eq. 4.3
V	volume	
V_o	equilibrium volume	Eq. 4.13
$\vec{v}$	particle velocity	p. 24
v_i	component of particle velocity	p. 26
$\vec{v}'$, v_i'	acoustic particle velocity	p. 26
$\vec{v}_o$	local steady value of $\vec{v}$	p. 24
W	power	
W_{ac}	acoustic (radiated) power	Eq. 1.1
W_{mech}	mechanical power	Eq. 1.1
W_{vibr}	vibratory power	Eq. 6.61
w	normal displacement of vibrating surface	p. 111
$\dot{w}$	vibratory velocity	p. 113
X, Y, Z	functions of x, y, z	Eq. 2.16
X	reactance	
X_r	radiation reactance	Eq. 3.1
x, y, z	cartesian coordinates	
x	general variable	
Y	Young's modulus	Eq. 5.1

Roman Letters (continued)

$\underline{Z}$	impedance	
$\underline{Z}_i$	input impedance	Eq. 5.114
$\underline{Z}_r$	radiation impedance	Eq. 3.1
$\underline{z}_a$	specific acoustic impedance	Eq. 2.68

Greek Letters

α, β	coefficients; parameters	
α	absorption or dissipation coefficient	Eq. 2.94
α_r	reflection coefficient	Eq. 2.105
α_t	transmission coefficient	Eq. 2.108
γ	ratio of specific heats	Eq. 4.27
∇	gradient operator	Eq. 1.36
∇^2	Laplacian	Eq. 1.40
$\partial\_$	partial derivative	
δ_{ij}	Kronecker delta	p. 12
η	variable	
η	loss factor	Eq. 2.90
η_{ac}	acoustic conversion efficiency	Eq. 1.1
η_{rad}	radiation efficiency; loss factor	Eq. 1.4
η_{vibr}	vibration conversion efficiency	Eq. 1.4
θ	angle; phase angle	
θ	temperature	
θ_i	angle of incidence	Fig. 2.2
λ	wave length	Eq. 2.8
μ, μ'	total mass per unit length, area	p. 113
ν	kinematic viscosity	Eq. 6.104
π	3.14159 . . .	
ρ	density	
ρ'	acoustic density fluctuation	Eq. 2.22
ρ_o	fluid static (mean) density	
ρ_s	density of structure material	p. 113
Σ	summation	
σ_r, σ_x	specific radiation resistance, reactance	Eq. 3.3
τ	time constant; time delay; decay time	
τ_{ij}	turbulent stress tensor	Eq. 3.14
Φ	amplitude of potential	Eq. 2.64
ϕ	potential	p. 14
ϕ	angle; phase angle	
ω	angular frequency $(2\pi f)$	Eq. 2.7

Abbreviations

AG	array signal-to-noise gain	p. 10
curl	vector rotation operator	Eq. 1.38
dB	decibel	p. 4
dBs	dB spectrum (1 Hz band)	p. 8
div	divergence operator	Eq. 1.37
cgs	centimeter-gram-second system of units	
a.c.	alternating current	
c.g.	center of gravity	
d.c.	direct current	
fps	feet per second	
grad	gradient operator	Eq. 1.35
Hz	Hertz (cycles per second)	
hp	horsepower	
kt	knot	
MKS	meter-kilogram-second system of units	
N	Newton (unit of force)	
Pa	Pascal (N/m^2)	
PWL	power level (dB)	Eq. 1.13
RP( )	real part of	Eq. 2.10
rms	root mean square	
rpm	revolutions per minute	
rps	revolutions per second	
SPL	sound pressure level (dB)	Eq. 1.11
TL	transmission loss (dB)	Eq. 1.9
W	Watt	
WWII	World War II	

A.2 Nomenclature by Chapter

Chapters 1-4

a	ratio of dp to $d\rho$	Eq. 2.22
$\bar{a}$	array parameter	Eq. 4.99
$\underline{c}$	complex speed of sound	Eq. 2.90
$D(\theta)$	directivity function	Eq. 4.66
DF	directivity factor	Eq. 4.102
DI	directivity index (dB)	Eq. 4.107
DT	detection threshold (dB)	p. 10
e	electric voltage	p. 4
$\vec{F}_g$	gravitational force	Eq. 2.36
$\vec{F}_p$	force due to gradient of pressure	Eq. 2.37
f_c	effective center frequency of a filter	Eq. 1.17

Chapters 1-4 (continued)

h_R, h_S	receiver and source depths	Fig. 4.12
I_o	reference intensity	Eq. 1.10
IL	intensity level (dB)	Eq. 1.10
K	spring constant for gas in a bubble	Eq. 4.26
k_o	wave number at resonance	p. 67
L, L'	array lengths	p. 83
M	momentum	Eq. 2.40
N	number of elements in an array	p. 83
N_{RD}	recognition differential (dB)	p. 10
P_i	incident pressure amplitude	Eq. 2.102
P_r, P_t	amplitudes of reflected and transmitted rays	Eq. 2.102
PTL	power transmission loss (dB)	Eq. 2.111
$\bar{p}$	pressure radiated by piston for $ka \ll 1$	p. 90
$\bar{p}_i$	rms pressure inside pipe	Eq. 4.136
p_{ij}	instantaneous stress tensor	Eq. 3.9
p_o	reference sound pressure	Eq. 1.11
R	effective cross-sectional radius	Fig. 4.20
R	resistance to bubble motion	Eq. 4.25
$\bar{r}$	rms distance	Eq. 4.53
r_H	horizontal distance	Fig. 4.12
r_o	reference distance	p. 43
SE	signal excess (dB)	Eq. 1.23
SL	signal level (dB)	Eq. 1.19
S/N	signal-to-noise ratio (dB)	p. 10
T_o	oscillation period	Eq. 4.31
TA	transmission anomaly (dB)	Eq. 2.87
$\bar{u}$	rms fluctuating fluid velocity in pipe	Eq. 4.136
W_o	reference power	Eq. 1.13
β	transmission parameter (Ch. 2)	Eq. 2.104
β	surface interference parameter (Ch. 4)	Eq. 4.54
δ	logarithmic decrement	Eq. 4.32
τ	time delay	Eq. 4.67
θ	phase angle between acoustic pressure and particle velocity	Eq. 2.80
θ_a	phase angle on surface of radiator	Eq. 4.5
θ_o	angle of maximum radiation	Eq. 4.64
θ_r, θ_t	reflection, transmission angles	Fig. 2.2
ϕ	array angle parameter	Eq. 4.89
ψ	phase angle between two sources	Eq. 4.57
ω_o	resonance angular frequency $(2\pi f_o)$	Eq. 4.28

Chapters 5 and 6

B, B_p	bending rigidity of beam, plate	p. 113; Eq. 6.2
b	beam element width	p. 111
c_B	irrotational compressional wave speed of bulk solids	Eq. 5.3
c_f	boundary-layer wall friction coefficient	Eq. 6.103
c_ℓ	longitudinal wave speed of solids	Eq. 5.1
c_p	plate wave speed	Eq. 5.9
c_s	shear wave speed	Eq. 5.2
D	boundary-layer velocity defect parameter (Sec. 6.6)	Eq. 6.102
E-B	Euler-Bernoulli	p. 116
$\underline{F}_f$	vibratory force transmitted to foundation	p. 143
$\underline{F}_i$	input force to mount generated by machine	p. 143
F_x	extensional force experienced by fiber of bending element	p. 111
F_z	normal force in bending	Fig. 5.5
f_c	coincidence frequency	p. 163
f_m	m-th order flexural resonance frequency	p. 124
H	vector potential for shear waves	Eq. 6.25
h	beam thickness; plate thickness	p. 120
h_p	plate thickness	Eq. 6.77
I	moment of inertia	p. 113
I'	mass moment of rotatory inertia	Eq. 5.20
I_i, I_t	intensity of incident and transmitted waves	Eq. 6.82
J_m	Lewis' coefficient for accession to inertia	Eq. 5.159
K	relative shear area	Eq. 5.23
K_p	relative shear area for plates	Eq. 6.3
k_B	beam flexural wave number	p. 177
k_c	wave number for coincidence	Eq. 6.35
k_c'	low-frequency approx. to wave number for coincidence	Eq. 6.58
k_f	wave number for flexural waves	p. 116
k_{f_o}	low-frequency approx. for flexural wave number	p. 126
k_o	wave number in fluid	p. 162
k_p	wave number for plate flexural waves	Eq. 6.76
k_s	wave number for shear waves	Eq. 6.25
k_t	effective hydrodynamic wave number (Sec. 6.6)	Eq. 6.119
L	beam length	p. 123
L_p	perimeter	Eq. 6.70
M	bending moment (Ch. 5)	p. 112
M_f	flexural wave Mach number	Eq. 6.38
m	order number of resonance; mode number	p. 124
N	number of modes	p. 142
n	mode number (circumferential)	Eq. 6.79
q_1	dynamic pressure of free-stream velocity	Eq. 6.116

Chapters 5 and 6 (continued)

R'_r	radiation resistance per unit area	Eq. 6.77
u	x-component of boundary-layer velocity (Sec. 6.6)	p. 185
u'	fluctuating x-component of boundary-layer velocity (Sec. 6.6)	Eq. 6.114
u_c	convection velocity for boundary-layer fluctuations	p. 193
u_θ	velocity at momentum thickness	p. 189
u^*_τ	boundary-layer friction velocity	Eq. 6.96
v_f	flexural wave speed	Eq. 5.34
v_{f_h}, v_{f_ℓ}	high- and low-frequency approx. to flexural wave speed	p. 118
W_i	input vibratory power	Eq. 5.116
w_o	vibratory displacement at source	p. 137
$\dot{w}_o$	input vibrational velocity	p. 137
X'_r	radiation reactance per unit area	Eq. 6.43
$\underline{Y}_{i,f,s}$	admittances of isolator, foundation and source (Sec. 5.9)	Eq. 5.149
$\underline{Z}_{i_\ell}$	low-frequency estimate of input impedance	Eq. 5.115
$\underline{Z}_{i,f,s}$	impedances of isolator, foundation and source (Sec. 5.9)	p. 143
z	distance from neutral plane (Ch. 5)	Fig. 5.4
z	distance from wall in boundary layer (Sec. 6.6)	p. 185
z_o	displacement of neutral plane	Fig. 5.15
z_τ	dimensionless distance from wall in boundary layer (Sec. 6.6)	Eq. 6.97
α'	relative rotatory inertia	Eq. 5.21
$\overline{\alpha}$	rotatory inertia relative to shear	Eq. 5.43
β	fluid loading factor	Eq. 6.45
Γ	shear parameter for flexural waves	Eq. 5.24
Γ_p	plate shear parameter	Eq. 6.4
γ	coefficient of bending wave exponential decay	p. 116
δ	bending wave shear-frequency parameter	Eq. 5.42
δ	boundary-layer thickness (Sec. 6.6)	p. 185
δ^*	boundary-layer displacement thickness	Eq. 6.94
ϵ	relative entrained mass	Eq. 5.17
ϵ	roughness height in boundary-layer theory (Sec. 6.6)	p. 189
η_r	radiation loss factor	Eq. 6.47
η_s	structural loss factor	Eq. 6.48
η_T	total loss factor	Eq. 6.48
θ	flexural angle in bending	Fig. 5.4
θ	boundary-layer momentum thickness (Sec. 6.6)	Eq. 6.95
θ_m	angle for maximum radiation	p. 172
θ_o	trace matching angle	Fig. 6.4
κ	radius of gyration	Eq. 5.15
λ_c	wave length at coincidence	
λ'_c	low-frequency calculated coincidence wave length	Eq. 6.68
λ_f	wavelength of flexural wave	Fig. 6.4

Chapters 5 and 6 (continued)

λ_o	wave length in fluid	Fig. 6.4
μ'_B, μ_p	beam and plate masses per unit area	Eq. 6.76
ν	plate curvature parameter	Eq. 6.78
ξ	extension	Fig. 5.4
ρ_s, $\overline{\rho}_s$	density of structure	p. 113
σ	Poisson's ratio	Eq. 5.4
$\overline{\sigma}_r$	specific radiation resistance for reverberant flexural wave field on plates	Eq. 6.69
τ_o	extrapolated wall shear stress (Sec. 6.6)	Fig. 6.19
τ_w	wall shear stress	Eq. 6.96
Φ	phase shift of flexural waves	Eq. 5.80
ϕ_m	modal amplitude	Eq. 5.134
Ω	reference angular frequency	Eq. 5.36
Ω_p	plate reference angular frequency	Eq. 6.9
ω_c	coincidence angular frequency	Eq. 6.34
ω'_c	low-frequency calculated angular frequency for coincidence	Eq. 6.45
ω_m	angular frequency for m-th resonance	p. 124

Chapters 7-10

A_p	piston cross-sectional area	Eq. 10.20
a	vortex core radius (Ch. 7)	p. 229
a	longitudinal separation of vortices (Ch. 9)	Fig. 9.5
$\overline{a}$	lift curve slope (Ch. 7)	Eq. 7.74
$\overline{a}$	mean reflection loss, in nepers (Ch. 8)	Eq. 8.36
$a(0)$	initial radius of growing or collapsing bubble	p. 206
a_c	bubble radius when collapse speed is maximum	Eq. 7.27
a_e	equilibrium radius for gas in a bubble	Eq. 7.17
a_m	minimum radius of a cavitation bubble	Eq. 7.20
a_o	maximum radius of a cavitation bubble	Eq. 7.10
B	number of blades	p. 258
b	thickness of airfoil, cylinder, etc.	Eq. 7.71
C_D	drag coefficient	Eq. 7.73
C_{D_F}	form drag coefficient	Eq. 9.49
C_F	force coefficient	Eq. 9.30
$\widetilde{C}_F$	fluctuating force coefficient	Eq. 9.27
C_L	steady-state lift coefficient	Eq. 7.72
$\widetilde{C}_L$	fluctuating lift coefficient	Eq. 9.36
C_p	pressure coefficient	Eq. 7.54
C_Q	torque coefficient	Eq. 8.19
C_T	steady-state thrust coefficient	Eq. 8.18
$\widetilde{C}_T$	fluctuating thrust coefficient	Eq. 9.33
c_f	boundary-layer wall friction coefficient	Eq. 7.58

Chapters 7-10 (continued)

E_{Kin}, E_{Pot}	kinetic and potential energies	p. 207
e	airfoil trailing edge thickness	Eq. 9.74
$\widetilde{F}$	fluctuating force	Eq. 9.1
F_D	drag force	Eq. 7.73
F_{D_F}	form drag	Eq. 9.48
F_L	lift force	Eq. 7.72
F_{L_i}	induced lift force	Eq. 9.55
$\widetilde{F}_o$	magnitude of fluctuating force	Eq. 9.1
$\widetilde{f}$	a.c. power-line frequency	Eq. 10.4
f_{CR}, f_i, f_p, f_x	piston forces (Sec. 10.4)	Fig. 10.5
$G(\gamma)$	Sear's function for airfoil gust response (Ch. 9)	Eq. 9.36
H	ocean depth (Sec. 8.7)	p. 281
H	effective piston depth (Sec. 10.4)	Eq. 10.32
h	lateral vortex spacing (Sec. 9.3)	Fig. 9.5
h	cavity depth (Sec. 9.6)	Fig. 9.15
I_f	intensity per 1 Hz band	Eq. 8.7
J	advance ratio	Eq. 8.21
j	number of engine cylinders (Sec. 10.4)	Eq. 10.36
K	cavitation parameter	Eq. 7.53
K_o, K_1	modified Bessel functions of second kind	Eq. 9.38
K_a	advance-speed cavitation parameter	Eq. 8.25
K_i	inception value of cavitation parameter	Eq. 7.55
K_t	tip cavitation parameter	Eq. 8.3
L_S'	overall source level above 100 Hz (dB)	p. 274
ℓ	mean free path for sound waves in ocean (Sec. 8.7)	Eq. 8.36
M_t	tip Mach number	Eq. 9.71
m	order of harmonic	Eq. 9.32
m_p	effective piston mass (Sec. 10.4)	Eq. 10.19
N	rotational speed in rpm	Eq. 8.1
N_i	rotational speed for cavitation inception (Ch. 8)	Eq. 8.1
N_i	number of inertia impacts per cycle (Sec. 10.4)	Eq. 10.36
n	rotational speed in rps	Eq. 8.2
P	static pressure differential for bubble growth or collapse	Eq. 7.5
$\overline{p}$	effective average piston pressure (Sec. 10.4)	Eq. 10.38
p_g	partial pressure of gas in bubble	Eq. 7.16
p_i	pressure inside a bubble	Eq. 7.1
p_i'	inductive near-field pressure (Ch. 9)	Eq. 9.11
p_m	maximum piston pressure (Sec. 10.4)	Eq. 10.20
p_v	vapor pressure	Eq. 7.16
Q	equilibrium partial gas pressure (Ch. 7)	Eq. 7.19
Q	propeller torque (Ch. 8)	Eq. 8.13
R	number of armature (rotor) slots or teeth (Sec. 10.2)	Eq. 10.4

Chapters 7-10 (continued)

R_N	Reynolds number	Eq. 7.56
S	piston stroke (Sec. 10.4)	Fig. 10.5
S_N	Strouhal number	Eq. 9.42
s	airfoil chord length (Ch. 7-9)	Eq. 7.71
s	piston linear speed (Sec. 10.4)	Eq. 10.33
s_i	image distance (Sec. 9.3)	Fig. 9.8
spc	strokes per cycle (Sec. 10.4)	Eq. 10.36
T	propeller thrust	Eq. 8.12
$\widetilde{T}$	fluctuating thrust	Eq. 9.33
T_c, T_g	time for bubble collapse and growth	p. 207
U_a	speed of advance	p. 261
U_s	free streamline velocity	Eq. 9.51
U_t	tip speed	Eq. 8.2
u_s	vortex sheet induced velocity (Sec. 9.3)	Eq. 9.45
W	weight of explosive in pounds (Sec. 7.10)	p. 243
W_N	Weber number	Eq. 7.57
$w(t)$	impact velocity (Ch. 10)	Eq. 10.5
w_i	propeller induced velocity (Ch. 8)	Fig. 8.7
w_o	gust amplitude (Ch. 9)	Eq. 9.36
w_o	peak value of impact velocity (Ch. 10)	Fig. 10.2
α	angle of attack	Eq. 7.74
α	ratio of inertia to pressure forces (Sec. 10.4)	Eq. 10.31
α_o	angle of attack for zero lift	Eq. 7.74
α_T	total effective absorption loss coefficient (Sec. 8.7)	Eq. 8.37
β	pitch angle (Ch. 8)	Fig. 8.7
β	connecting-rod angle (Sec. 10.4)	Fig. 10.5
Γ	circulation	Eq. 7.64
γ	Sear's reduced frequency parameter (Ch. 9)	Eq. 9.37
δ, δ°	density of ships on ocean surface (Sec. 8.7)	Eq. 8.40
δ	piston clearance (Sec. 10.4)	Fig. 10.5
η_i, η_p	propeller efficiencies (Ch. 8)	Eqs. 8.20, 8.23
η	viscosity	p. 209
ρ_p	density of piston material (Sec. 10.4)	Eq. 10.32
σ	surface tension	Eq. 7.1
τ	thrust loading factor (Ch. 8)	Eq. 8.24
τ_w	wall shear stress	Eq. 7.58
Φ	hydrodynamic potential	Eq. 7.2
ϕ	hydrodynamic pitch angle (Ch. 8)	Fig. 8.7
ω	angular speed $(2\pi n)$	Eq. 9.40

A.3 Abbreviations Used in References

A.S.A.	Acoustical Society of America
A.S.M.E.	American Society of Mechanical Engineers
ASW	Anti-Submarine Warfare
C.I.T.	California Institute of Technology
D.T.M.B.	David Taylor Model Basin
J.	Journal of
M.I.T.	Massachusetts Institute of Technology
N.A.C.A.	National Advisory Committee for Aeronautics
N.A.S.A.	National Aeronautics and Space Administration
N.D.R.C.	National Defense Research Committee
N.S.R.D.C.	Naval Ship Research and Development Center
O.N.R.	Office of Naval Research (U.S. Navy)
O.S.R.D.	Office of Scientific Research and Development
R. and M.	Reports and Memoranda
S.A.E.	Society of Automotive Engineers
U.C.L.A.	University of California at Los Angeles
U.S.N.	United States Navy
op. cit.	Work cited in references for *this chapter*

APPENDIX B

DECIBEL ARITHMETIC

Decibels are especially useful when numbers representing physical quantities are being multiplied or divided. As indicated in Section 1.2, these operations are replaced by addition and subtraction. However, decibels are awkward to use when addition or subtraction of physical quantities is required. The method of accomplishing the indicated process depends on whether the signals are coherent or incoherent. Formulas for carrying out the several types of decibel addition and subtraction are given in this appendix.

Addition and Subtraction of Coherent Signals

Consider two instantaneous acoustic pressures having the same frequency,

$$p'_1 = p_1\sqrt{2}\cos(\omega t - \theta_1) \tag{B.1}$$

and

$$p'_2 = p_2\sqrt{2}\cos(\omega t - \theta_2)\ , \tag{B.2}$$

where p_1 and p_2 are rms pressures. The corresponding sound pressure levels are

$$L_1 = 20 \log \frac{p_1}{p_o} \text{ and } L_2 = 20 \log \frac{p_2}{p_o} , \tag{B.3}$$

where p_o is the reference pressure, as discussed in Section 1.2. The result of combining these signals is

$$L_T = 10 \log \frac{p_T^2}{p_o^2} , \tag{B.4}$$

where

$$p_T^2 = \overline{(p_1' + p_2')^2} = p_1^2 + p_2^2 + 4p_1p_2 \overline{\cos(\omega t - \theta_1)\cos(\omega t - \theta_2)}$$

$$= p_1^2 + p_2^2 + 2p_1p_2 \cos(\theta_1 - \theta_2) . \tag{B.5}$$

This result depends on the phase angle between the two signals. If they are equal and in phase, the result is 6 dB higher than either. If they are equal and exactly out of phase, the resultant pressure is zero. If they are not equal in magnitude, the maximum value of L_T will always be less than 6 dB above the level of the stronger signal. Equation B.5 applies to subtraction as well as addition since these are equivalent processes but with different phase angles.

Incoherent Addition

Most often the signals being considered are either broadband or of different frequencies, in which case the long-term average of the product of the cosines is zero. In this case,

$$L_T = 10 \log \frac{p_1^2 + p_2^2}{p_o^2} = 10 \log \left(10^{L_1/10} + 10^{L_2/10}\right) . \tag{B.6}$$

This is known as the *power sum* of two levels and can be extended to any number of incoherent sources.

A rapid method of making power summations can be derived from the above. Taking $L_1 \geqslant L_2$, Eq. B.6 can be written

$$L_T = 10 \log \frac{p_1^2}{p_o^2} \left(1 + \frac{p_2^2}{p_1^2}\right) = L_1 + 10 \log \left(1 + \frac{1}{10^{(L_1 - L_2)/10}}\right) . \tag{B.7}$$

It follows that unless L_2 is within 12 dB of L_1 the total level will be virtually unaffected. When L_2 is within 12 dB of L_1, Fig. B.1 can be used to obtain the power sum.

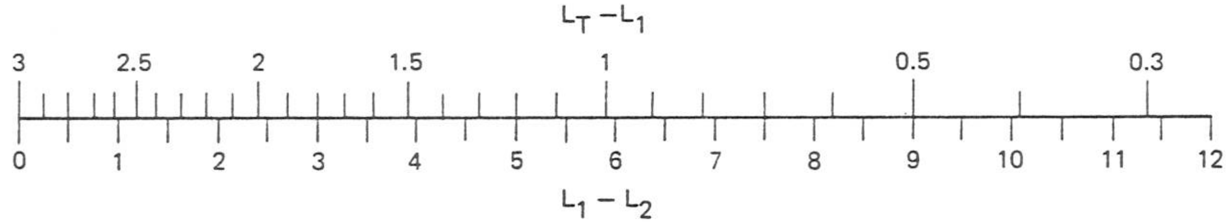

Fig. B.1. Power Sum of Two Incoherent Pressures

Subtraction of Incoherent Sources

When one source, L_1, is turned off, the level will drop by an amount that will depend upon how close the level of the removed source was to the total from all sources. Solving Eq. B.7 for two incoherent sources for L_2 gives

$$L_2 = L_T - 10 \log \left[\frac{10^{(L_T - L_1)/10}}{10^{(L_T - L_1)/10} - 1} \right] = L_T + 10 \log \left[1 - \frac{1}{10^{(L_T - L_1)/10}} \right] . \tag{B.8}$$

This result is graphed in Fig. B.2, where it can be seen that the remaining signal becomes lower as the secured source approaches the total. When the two sources are equal, the difference level is 3 dB lower than their sum, in agreement with Fig. B.1.

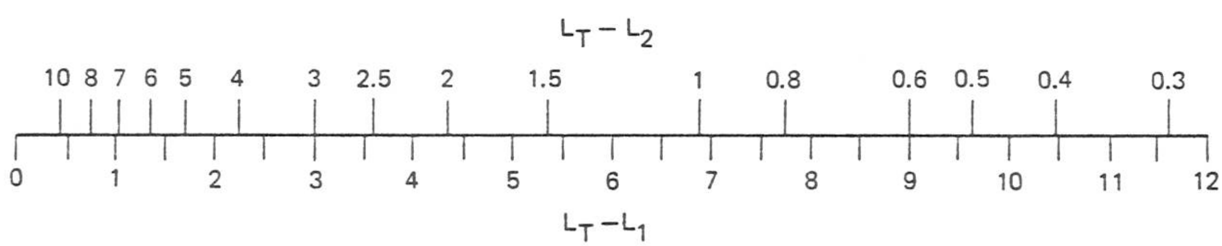

Fig. B.2. Incoherent Subtraction of Pressures