CHAPTER 9

RADIATION BY FLUCTUATING-FORCE (DIPOLE) SOURCES

Sources that produce fluid volume fluctuations are dominant when they exist. There are many cases, however, in which they are absent and in which force fluctuations produce the strongest sounds. Fluctuating, or unsteady, forces occur as by-products of steady work-producing forces, as discussed in Chapter 1. When such forces cause a structure to vibrate, radiation occurs from flexural waves, as covered in Chapter 6. When forces are hydrodynamic in origin, sound is radiated directly into the fluid independent of any motion of a fluid boundary. This rigid-body, oscillating-force radiation is of dipole nature, as discussed in Section 3.4. The present chapter deals primarily with dipole-type radiation of sound by hydrodynamic fluctuating forces acting on rigid bodies. All lifting surfaces radiate sound in this way, and this mechanism accounts for much of the sound generated by hydraulic machines such as turbines, pumps and propellers as well as by fans, blowers and compressors.

9.1 Dipole Sound Sources

There are a number of ways in which dipole sound fields can be generated. Physically, the sources usually involve fluctuating forces or oscillating motion of a rigid body; mathematically, they can be expressed in terms of two equal out-of-phase monopole volume sources or by the spatial derivative of a monopole field.

Acoustic Field of a Concentrated Force

Just as large fluctuating-volume sources can be treated by summing the effects of many monopoles, so also sound radiation from large bodies experiencing fluctuating forces can be calculated by integrating the sound fields of elemental concentrated-force radiators. The sound field of a concentrated force varying arbitrarily with time was derived in Chapter 3. Since superposition may be assumed, any arbitrary force may be decomposed into harmonic components using Fourier's theorem as discussed in Section 1.5. Assuming a sinusoidally varying force, Eq. 3.31 for the radiated acoustic pressure due to a concentrated force becomes

$$\underline{p}' = \frac{\dot{\underline{F}} \cos\theta}{4\pi r c_o} e^{-ikr} = \frac{i\omega \underline{\tilde{F}}_o \cos\theta}{4\pi r c_o} e^{i(\omega t - kr)} \quad . \tag{9.1}$$

This result is applicable only if the dimensions of the body or surface experiencing the force are small compared to an acoustic wavelength, and if the distance r to the field point is large compared to a wavelength. The cosine radiation pattern is characteristic of a dipole.

The acoustic intensity of an elementary force dipole is given by

$$I(\theta) = \frac{\overline{p'^2}}{\rho_o c_o} = \frac{\omega^2 \overline{\tilde{F}^2}}{16\pi^2 r^2 \rho_o c_o^3} \cos^2 \theta \ . \qquad (9.2)$$

Integrating over a sphere, the average intensity is

$$\bar{I} = I(0^o) \frac{1}{4\pi} \int_0^{2\pi} \int_0^{\pi} \cos^2 \theta \sin \theta \, d\theta d\phi = \frac{1}{3} I(0) \ , \qquad (9.3)$$

from which it follows that the acoustic power is

$$W_{ac} = \frac{\omega^2 \overline{\tilde{F}^2}}{12\pi \rho_o c_o^3} \ . \qquad (9.4)$$

Comparing this expression to Eq. 4.17 for the power radiated by a monopole fluctuating-volume source, its dipole character is indicated by the cubic dependence on speed of sound, which implies a cubic dependence of radiation efficiency on Mach number.

Oscillating Rigid Sphere

As discussed in Chapter 3, it is the fluctuating surface-pressure field associated with a fluctuating force that radiates scund. Another way of producing fluctuating surface pressure is by oscillating motion of a rigid sphere, as depicted in Fig. 9.1. Consider a rigid sphere of radius a_o

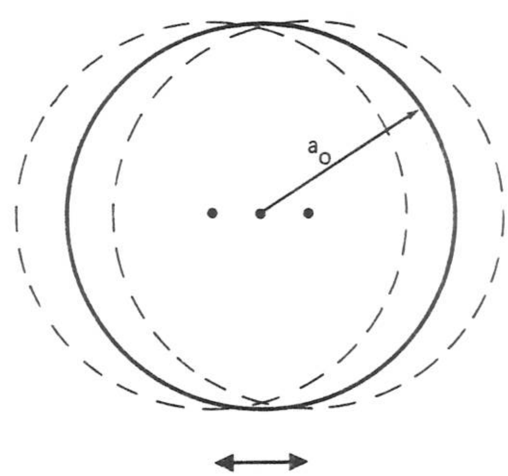

Fig. 9.1. Translational Oscillation of a Rigid Sphere

executing simple harmonic motion with peak speed u_o. The radial component of speed normal to the surface at each point on the sphere is given by

$$\underline{u}(\theta) = u_o \cos\theta\, e^{i\omega t} \quad , \tag{9.5}$$

where θ is the angle between the radius vector and the direction of motion. Equating this surface velocity to the radial acoustic particle velocity and then solving for the far-field acoustic pressure, one finds

$$\underline{p}' \doteq - \frac{\omega \rho_o S_o u_o}{4\pi r} \left(\frac{ka_o}{2} \cos\theta \right) e^{i(\omega t - kr)} \quad , \tag{9.6}$$

provided $ka_o << 1$.

The total power radiated by an oscillating sphere is

$$W_{ac} = \frac{1}{3}\, 4\pi r^2 \frac{\overline{p'^2(0)}}{\rho_o c_o} = \frac{(ka_o)^4}{24} \rho_o c_o S_o u_o^2 \quad , \tag{9.7}$$

from which it follows that the radiation resistance, Eq. 3.2, is

$$R_r = \frac{W_{ac}}{\frac{1}{2} u_o^2} = \frac{1}{12} \rho_o c_o S_o (ka_o)^4 \quad , \tag{9.8}$$

and the specific radiation resistance, Eq. 3.3, is

$$\sigma_r = \frac{R_r}{\rho_o c_o S_o} = \frac{1}{12} (ka_o)^4 \quad . \tag{9.9}$$

This result is identical to that given by Eq. 3.18 for a first-order multipole, as derived from a general spherical source.

Not only do dipoles differ from monopoles in having cosine directional patterns, but also they differ in the nature of their near-field pressures. In the case of a pulsating volume source, a single expression for pressure decreasing with distance from the center applies at all locations outside the source. This is not the case for dipoles. A large fraction of the power that is required to oscillate a rigid sphere goes into a near-field hydrodynamic sloshing motion of the fluid that does not radiate as sound. For small ka_o and small kr, the pressure near an oscillating rigid sphere is

$$\underline{p}_i' \doteq i \frac{\omega \rho_o S_o u_o}{4\pi r^2} \left(\frac{a_o}{2} \cos\theta \right) e^{i(\omega t - kr)} \quad , \tag{9.10}$$

which, being inductive, decays as the square of the distance from the center of the sphere. Comparing Eq. 9.10 for the inductive field to Eq. 9.6 for the acoustic field, we find

$$\underline{p}_i' = \frac{\underline{p}'}{ikr} \quad . \tag{9.11}$$

Hence, the inductive field lags the acoustic field by 90° at each radial distance. The total fluctuating pressure at any radius is the sum of the two components.

The reactive force associated with the fluid motion involved in the oscillation of a rigid sphere equals the integral of the component of the pressure in the direction of motion over the surface,

$$\underline{F}_x = \int_{S_o} \underline{p}_i'(a_o) \cos\theta \, dS \; . \tag{9.12}$$

Using Eq. 9.10 with $r = a_o$ for the surface pressure, and carrying out the indicated integration,

$$\underline{F}_x = i \frac{\omega \rho_o S_o u_o e^{i\omega t}}{8\pi a_o} \int_o^{2\pi} \int_o^{\pi} a_o^2 \cos^2\theta \sin\theta d\theta d\phi$$

$$= \frac{i}{6} \rho_o c_o (ka_o) S_o u_o e^{i\omega t} \; . \tag{9.13}$$

The reactive component of the radiation impedance equals the reactive force divided by the velocity. It follows that

$$X_r = \frac{F_x}{u_o e^{i\omega t}} = \frac{1}{6} \rho_o c_o S_o (ka_o) \; , \tag{9.14}$$

which is the reactance of a mass equal to one half of that displaced by the sphere.

For small ka_o the radiation resistance given by Eq. 9.8 is small compared to the reactance and the impedance can be taken to be entirely reactive. It follows that the radiation efficiency is

$$\eta_{rad} = \frac{R_r}{|Z_r|} \doteq \frac{R_r}{X_r} = \frac{1}{2} (ka_o)^3 \tag{9.15}$$

in complete agreement with Eq. 3.23.

The equivalence of an oscillating rigid sphere and a fluctuating force as sound sources can be further demonstrated by expressing the acoustic field of an oscillating sphere in terms of the force required to keep it in motion. This force is composed of two terms: the reactive force of the medium, as given by Eq. 9.13, and the force required to accelerate its mass. Assuming that the sphere has the same density as the fluid, the total force is

$$\underline{\widetilde{F}} = \underline{F}_x + m\dot{u} = \underline{F}_x + i\omega\rho_o \left(\frac{4}{3} \pi a_o^3 \right) u_o e^{i\omega t} \; , \tag{9.16}$$

from which it follows that

$$\underline{\widetilde{F}} = i\rho_o c_o S_o \left(\frac{ka_o}{2} \right) u_o e^{i\omega t} \; . \tag{9.17}$$

Equation 9.6 for the radiated pressure may be written

$$\underline{p}' \doteq \frac{i\omega\widetilde{F}}{4\pi r c_o} \cos\theta\, e^{-ikr} \,, \tag{9.18}$$

which is equivalent to Eq. 9.1. Thus, the two source expressions are equivalent, provided the wavelength is large compared to the diameter of the sphere.

Spheres Pulsating Out of Phase

Dipole sound patterns are also generated by two equal out-of-phase monopoles separated by a distance small compared to a wavelength, as was developed in Section 4.5. If each of the pulsating spheres comprising the dipole has source strength given by

$$\underline{Q} = Q_o\, e^{i\omega t} \,, \tag{9.19}$$

and their centers are separated by d, then from Eq. 4.75

$$\underline{p}' = i\,\frac{kQ_o d}{4\pi r} \cos\theta\, e^{-ikr} = -\,\frac{\omega^2 Q_o d}{4\pi r c_o} \cos\theta\, e^{i(\omega t - kr)} \,, \tag{9.20}$$

where $\cos\theta$ replaces $\sin\theta$ because of the present choice of the dipole axis as the reference for θ rather than a perpendicular.

The product $Q_o d$ is the magnitude, or strength, of the dipole, represented by D_o in the following equations. Comparing Eq. 9.20 to Eqs. 9.1 and 9.6, dipole strength can be related to the strength of a fluctuating force and to the size and velocity of an oscillating sphere by

$$D_o \equiv Q_o d = \frac{\widetilde{F}_o}{i\omega} = \rho_o S_o u_o \frac{a_o}{2} \,. \tag{9.21}$$

General expressions for the acoustic pressure, intensity and power of dipoles are

$$\underline{p}' = -\,\frac{\omega^2 D_o}{4\pi r c_o} \cos\theta\, e^{i(\omega t - kr)} \,, \tag{9.22}$$

$$I = \frac{\overline{p'^2}}{\rho_o c_o} = \frac{\omega^4 D_o^2}{32\pi^2 r^2 \rho_o c_o^3} \cos^2\theta \,, \tag{9.23}$$

and

$$W_{ac} = \frac{4}{3}\pi r^2 I(0) = \frac{\omega^4 D_o^2}{24\pi \rho_o c_o^3} \,. \tag{9.24}$$

All of these expressions written in terms of dipole strength reduce to the corresponding equations in terms of force or velocity flux, using the relations of Eq. 9.21.

Dipole Fields from Monopole Fields

The complete pressure field of a dipole, including both inductive and acoustic components, can be expressed as a spatial derivative of a monopole pressure field. Using Eq. 4.15 for the pressure field of a monopole,

$$dp'_o = \frac{\partial p'_o}{\partial x} dx = \frac{\partial p'}{\partial r} \frac{\partial r}{\partial x} dx = \frac{i\omega Q_o e^{i\omega t}}{4\pi} \frac{\partial}{\partial r} \left(\frac{e^{-ikr}}{r} \right) \left(\frac{\partial r}{\partial x} \right) dx$$

$$= \frac{i\omega Q_o}{4\pi r} \left(-\frac{1}{r} - ik \right) \left(\frac{\partial r}{\partial x} \right) e^{i(\omega t - kr)} dx \,. \tag{9.25}$$

Interpreting $\partial r/\partial x$ as $\cos\theta$ and equating dx to the separation of the poles, d, we find

$$dp'_o = \frac{\omega^2 Q_o d}{4\pi r c_o} \left(1 - \frac{i}{kr} \right) \cos\theta \, e^{i(\omega t - kr)} \,, \tag{9.26}$$

which is the negative of the complete dipole pressure field obtained by adding the acoustic component, Eq. 9.20, to the inductive component, Eq. 9.11.

This relation between dipole pressure fields and monopole fields can be generalized. Quadrupole fields can be obtained from derivatives of dipole fields, or from second derivatives of monopole fields. Higher order fields are produced by higher order derivatives.

9.2 Propeller Blade Tonals

Blade tonal radiation from non-cavitating propellers and from fans, compressors, turbines and pumps is an important noise source. The initial analysis of this problem by Gutin (1936) was concerned with sound radiation by a rotating static force distribution, i.e., sound radiation by a finite-bladed impeller in a uniform inflow, for which thrust and torque are constant. As discussed in Section 3.5, Gutin found that the sound radiated depends very strongly on both the number of blades and tip Mach number. At the low Mach numbers of marine propellers and of most hydraulic machinery Gutin noise is negligible compared to that generated as a result of force fluctuations; further discussion of his analysis will therefore be omitted. Readers interested in this subject are referred to Blokhintsev (1946) and to Richards and Mead (1968).

General Oscillating Hydrodynamic Force

When oscillating force is generated in connection with fluid flows, it is useful to define an oscillating force coefficient, $\widetilde{C}_F$, analogous to the lift and drag coefficients defined by Eqs. 7.72 and 7.73. Thus, writing

$$\widetilde{C}_F \equiv \frac{\widetilde{F}_{rms}}{\frac{1}{2} \rho_o U_o^2 S} \,, \tag{9.27}$$

the intensity and radiated power of a concentrated fluctuating force may be expressed by

$$I(\theta) = \frac{\rho_o U_o^3 S}{16 r^2} \left(\frac{f^2 S}{U_o^2} \right) \left(\frac{U_o}{c_o} \right)^3 \widetilde{C}_F^2 \cos^2\theta \tag{9.28}$$

and

$$W_{ac} = \frac{\pi}{12} \rho_o U_o^3 S \left(\frac{f^2 S}{U_o^2} \right) \widetilde{C}_F^2 M^3 \quad . \tag{9.29}$$

These equations, which may be used in place of Eqs. 9.2 and 9.4, show dependence on the sixth power of the flow speed and on the square of a dimensionless frequency coefficient. Calculation of the sound power radiated by various types of oscillating force requires expressions for the oscillating force coefficient and for the dimensionless frequency.

Usually oscillating forces are unwanted by-products of steady work-producing forces. The mechanical power associated with a steady force can be written

$$W_{mech} = \frac{1}{2} \rho_o U_o^3 S C_F \quad . \tag{9.30}$$

Combining this expression with Eq. 9.29, the acoustic efficiency is

$$\eta_{ac} \equiv \frac{W_{ac}}{W_{mech}} = \frac{\pi}{6} \left(\frac{f^2 S}{U_o^2} \right) \left(\frac{\widetilde{C}_F}{C_F} \right)^2 M^3 C_F \quad . \tag{9.31}$$

Noise from Oscillating Thrust

As discussed in Section 8.4, operation of a propeller in a wake having circumferential variations results in oscillating components of thrust at multiples of the blade frequency, which are given by

$$f = mBn \quad . \tag{9.32}$$

Here m is the order of the harmonic, B the number of blades and n the rotational speed in rps. Defining an rms oscillating thrust coefficient, $\widetilde{C}_T$, in a manner analogous to that of Eq. 8.18 for the steady thrust, by

$$\widetilde{C}_T \equiv \frac{\widetilde{T}_{rms}}{\rho_o n^2 D^4} \quad , \tag{9.33}$$

the intensity of directly radiated sound at the mth harmonic is then given from Eq. 9.2 by

$$I_m(\theta) \doteq \frac{m^2 B^2}{4 r^2 c_o^3} \rho_o n^6 D^8 \widetilde{C}_{T_m}^2 \cos^2 \theta \quad . \tag{9.34}$$

Thus, the intensity increases as the sixth power of tip rotational speed, as disk area, and as the square of number of blades, harmonic number and oscillating thrust coefficient. From Eq. 9.4 and the definition of the steady-state thrust coefficient given by Eq. 8.18, the acoustic conversion efficiency can be expressed by

$$\eta_{ac} = \frac{m^2 B^2}{3\pi^2 J} \frac{\widetilde{C}^2_{T_m}}{C_T} \left(\frac{\pi n D}{c_o}\right)^3 . \tag{9.35}$$

For typical propeller tip speeds of the order of 25 to 50 m/sec and oscillating thrust coefficients of 2 to 10% of that for the steady-state thrust, acoustic conversion efficiencies for direct radiation of blade-rate tonals are estimated to be between 10^{-10} and 10^{-8}, well below that for propeller cavitation.

While not efficient radiators of blade tonal components at the low Mach numbers of marine propellers, blade-rate forces transmitted by propeller shafts are a dominant cause of hull vibration. When these vibratory forces coincide with low-frequency hull resonances of the types discussed in Sections 4.11 and 5.11, severe vibrations may occur. Not only are these vibrations harmful to men and machines but also they may radiate blade tonal components at higher levels than those radiated directly by the propeller.

Factors Affecting Oscillating Thrust

Primarily because of the interest of naval architects in reducing shipboard vibrations and preventing propeller shaft fatigue failures, considerable effort has been devoted by a number of investigators to the development of methods of calculating propeller alternating thrust. The calculation of steady-state propeller forces for uniform inflow conditions is itself a complex problem for which no single method has proven to be clearly superior. Non-steady effects caused by wake operation make the problem even more untractable. As summarized by Stern and Ross (1964), the various published methods may be classified as either quasi-steady or unsteady and either two-dimensional or three-dimensional. In quasi-steady methods the forces on a propeller blade are calculated at each angular position as though the wake at that position existed over the entire 360°. In unsteady methods, unsteady airfoil theory developed by von Karman and Sears (1938) is used in deriving oscillating lift coefficients for blade elements. In two-dimensional analyses, induced velocity components due to helical trailing vortex sheets, cascade effects of other blades, tip vortices and other three-dimensional effects are ignored. Brown (1964) compared the various methods, finding, as shown in Fig. 9.2, that quasi-steady analysis is in good agreement with his three-dimensional unsteady theory when calculating the fundamental and second harmonic components while a two-dimensional unsteady method gives good results for the fourth and higher harmonics.

According to two-dimensional unsteady airfoil theory, the oscillatory lift experienced by an airfoil in a sinusoidal gust of amplitude w_o is given by

$$\underset{\sim}{\widetilde{C}}_L = 2\pi \frac{w_o}{U_o} e^{i\omega t} \underline{G}(\gamma) , \tag{9.36}$$

where

$$\gamma \equiv \frac{\omega s}{2U_o} \tag{9.37}$$

is called the *reduced frequency* of the gust. The phase angle of the function $\underline{G}(\gamma)$ depends upon the reference point on the airfoil. When expressed relative to the center as originally done by Sears (1949), the phase varies greatly, but Brown (1964) and Lowson (1970) have shown that the phase

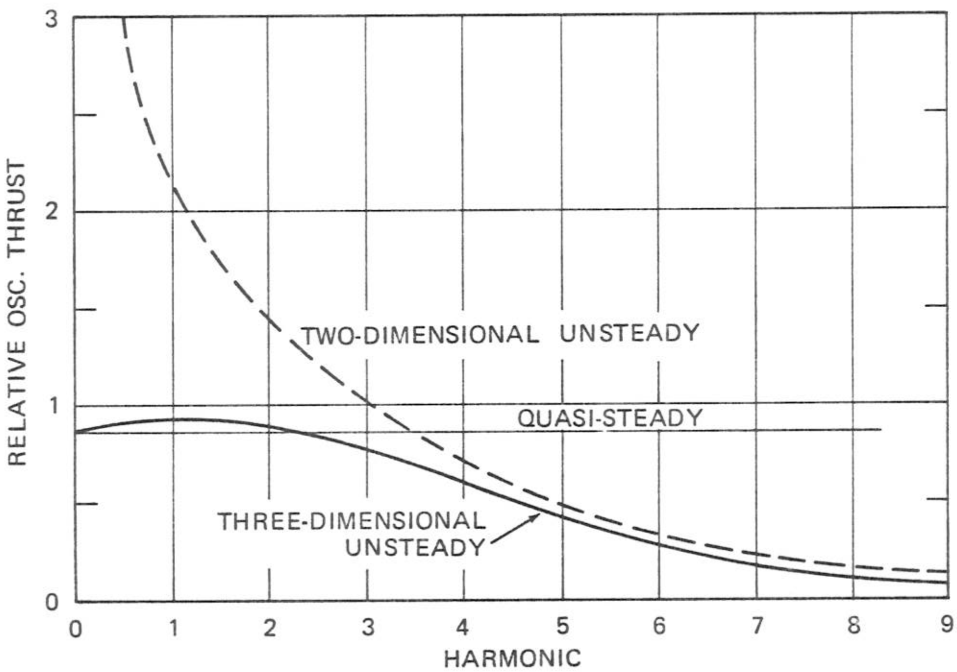

Fig. 9.2. Comparison of Computational Methods for Propeller Fluctuating Thrust, after Brown (1964)

angle varies by less than 45° over the entire frequency range when the leading edge is used as the reference point. As expressed by Brown,

$$\underline{G}_{LE}(\gamma) = \frac{e^{-i\gamma}}{i\gamma\left[K_o(i\gamma) + K_1(i\gamma)\right]} , \tag{9.38}$$

where K_o and K_1 are modified Bessel functions of the second kind. This function is plotted in Fig. 9.3. For the high values of the reduced frequency typical of marine propellers, especially for the higher harmonics, Eq. 9.38 reduces to

$$\underline{G}_{LE}(\gamma) \doteq \frac{1}{\sqrt{2\pi\gamma}} \, e^{-i(\pi/4)} , \tag{9.39}$$

which relation is valid within 3% for $\gamma \geqslant 1$.

The procedure used when making unsteady airfoil calculations is to perform an harmonic analysis of the circumferential wake at each radial station. Since the component of the wake having periodicity equal to m times B is the only component causing the mth harmonic of the blade-rate oscillating thrust, the complex oscillating force is computed for this component at each radial station and then integrated over the radius.

It is apparent from this discussion that number of blades is a most important factor affecting oscillating thrust. Some wakes, such as that shown in Fig. 8.12, have strong components at certain

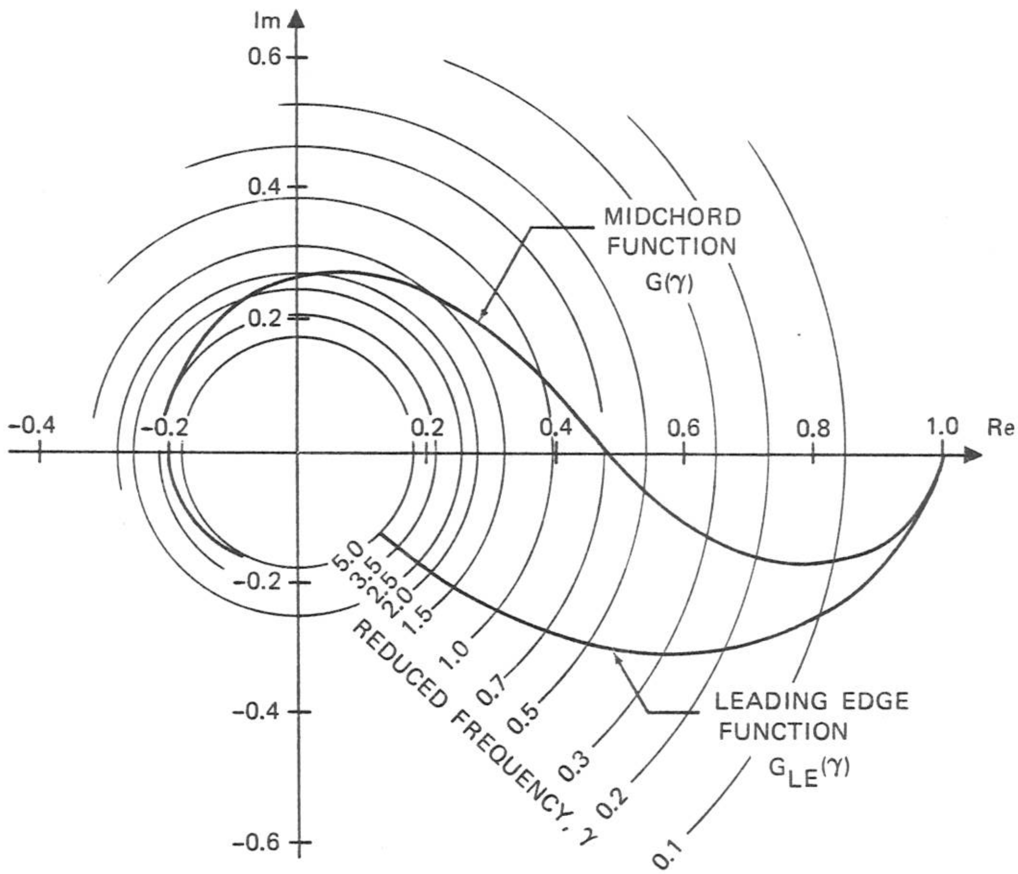

Fig. 9.3. Sears Function Relative to Midchord and to Leading Edge, after Brown (1964)

even harmonics. Selection of an odd number of blades may result in a much lower fluctuating force than would selection of an even number. In other cases, such as for the wake depicted in Figs. 8.10 and 8.11, there is little difference between four-, five- or six-bladed propellers.

Another factor affecting the fluctuating thrust is the shape of the leading edge. Cox and Morgan (1972) have demonstrated that skewing the leading edge tends to reduce the net force by causing the various radial sections to experience their forces out of phase with each other. Skewing also has the effect of increasing the section chord length, thereby increasing γ and slightly reducing the oscillating force, as implied by Eq. 9.39.

Calculations of oscillating thrust for a given propeller show relatively small variations of $\widetilde{C}_T$ with advance ratio J. It follows from Eq. 9.35 and Fig. 8.8 that somewhat lower acoustic efficiencies, i.e., lower blade tonals, will be achieved if propellers are operated at advance ratios slightly lower than optimum from the point of view of efficiency.

Propeller-Induced Hull Forces

In addition to direct excitation of hull vibrations by oscillating thrust transmitted through the propeller shaft and thrust bearing to the hull, the rotating pressure field associated with a propeller can induce oscillating forces on nearby hull plating, fins and/or struts. As previously discussed, direct radiation from rotating pressure fields is very small at the low Mach numbers of marine propellers. However, inductive near-field pressures can be significant and the forces experienced by nearby stationary surfaces can be large. In a summary of this subject, Breslin (1962) noted that important contributions to the inductive pressure field are made by both blade thickness and lift distributions and that propellers in non-uniform wakes produce larger forces. As confirmed by Tsakonas et al (1964), induced pressures and radiated sound both decrease markedly as tip clearances and/or number of blades are increased. The decrease of induced pressure fields experienced with increasing numbers of blades is a contributing factor in the present trend toward the use of five- and six-bladed propellers on modern high-power merchant ships.

Submarine and torpedo hulls also experience oscillating forces transmitted by propeller induction pressure fields. However, Chertock (1965) found these induced forces to be only of the order of 6 to 10% as large as those transmitted directly by the shaft. Tsakonas and Breslin (1965) also calculated the longitudinal force induced by a propeller on a prolate spheroid and found this force to vary inversely with the length-to-diameter ratio of the body. They noted that transverse forces induced in fins augment hull vibrations.

The importance of struts in transmitting oscillating forces to ship hulls was recognized by Lewis in the 1930's. More recently, Pinkus et al (1963) have calculated the force induced in a finite plate, representing an appendage, by a vortex distribution representing a blade moving past one end; and Lewis (1963, 1967) has reported on measurements of strut oscillating forces made in a water tunnel. Both studies show a very rapid decrease of induced force with increasing clearance, leading to the conclusion that this problem can be eliminated by giving attention to the spacing between propeller and stationary surfaces such as struts, fins and rudders.

Blade-Vortex Interaction Noise

Occasionally marine propellers emit a sound that resembles what one would expect from repeated slaps occurring at blade frequency. It is believed that the source of such *blade-slap noise* is the passage of blades through a vortex. Vortices originating from the tips of lifting surfaces upstream of the propeller, or from the first propeller of a counterrotating pair, may pass through the propeller disk. Each time a blade comes close to a vortex it experiences a force much as from an abrupt gust. Each force pulse generates a sound pulse, resulting in pulses repeated at blade frequency. The phenomenon has been studied by Simons (1966) and Widnall (1971) in connection with helicopter blade-slap noise. They found that the sound radiated is not only a function of the strength of the vortex but also of its core size levels, being lower for larger cores.

Shaft-Rate Components

Tonal components at shaft rate also occur in propeller spectra, though at much lower levels than blade tonals. Their presence implies modulation of the blade tonals at shaft rotational frequency. Such modulation would be expected to occur either if the blades were not mechanically identical and/or if their spacings were not exactly equal. Furthermore, even if blades and spacings were the same, forces experienced by the individual blades could differ somewhat since blade flexural vibrations do not occur identically. Expressing a modulated blade harmonic by

$$p = P\left(1 + \sum_j \alpha_j \cos j\omega t\right) \cos mB\omega t \qquad (9.40)$$

leads to

$$p = \frac{P}{2} \sum_j \alpha_j \left[cos\,(mB + j)\omega t + cos\,(mB - j)\omega t \right] , \qquad (9.41)$$

which series represents a family of tonals at multiples of the shaft rotational frequency.

Rotor-Stator Interactions

When center-line propellers are used on bodies of revolution such as torpedoes or submarines, the major cause of wake variation is the fins. In this case, oscillating forces may be considered to result from *rotor-stator interaction*. Similar rotor-stator interactions are also dominant sources of tonal noise from fans, compressors, pumps and turbines, and the subject has received considerable attention in connection with noise from such machinery.

Kemp and Sears (1953, 1955) applied unsteady airfoil theory developed by von Karman and Sears (1938) and Sears (1941) to the rotor-stator interaction problem, finding that potential interaction effects and viscous wake effects are often of the same order of magnitude. For close spacing of rotor and stator, potential interaction effects dominate. These forces, which are similar to appendage forces previously discussed, decrease quite rapidly with increased spacing. On the other hand, viscous effects caused by wakes from the first row interacting with the second row decay gradually with distance. Kemp and Sears found viscous effects to be proportional to the drag coefficient of the upstream airfoils. Later studies by Sharland (1964), Fincher (1966), Lowson (1968) and Morfey (1970) confirmed these conclusions. Mather et al (1971) reported that the levels of shaft-rate tonals are also influenced by rotor-stator interaction effects. Hanson (1973, 1974) found little difference in the spectra whether the stator precedes or follows the rotor.

Blade tonals give a distinctive character to rotor noise which is often quite annoying to humans. Mellin and Sovran (1970), Duncan and Dawson (1975) and others have proposed that unequal rotor and/or stator spacings be used in order to reduce blade-frequency tonals. Shaft-rate tonals are thereby increased and the total sound output remains the same, but the broadband characteristic of the resultant sound is more acceptable.

Blade-Turbulence Interactions

Not only do rotors interact with the wakes of upstream stators, producing blade and shaft rate tonals, but also they may interact with turbulent inflow velocity fluctuations and thereby produce a broadband noise spectrum. The importance of inlet turbulence was demonstrated by Sharland (1964), who found an 8 dB increase in noise from a fan operating in a turbulent inflow compared to its operation in a smooth flow. Further study of this noise source by Mani (1971) and Mugridge (1973) showed its importance for values of the Sears parameter, γ, defined by Eq. 9.37, up to about 10. Higher-frequency broadband noise is associated with boundary-layer turbulence and with turbulent eddies in blade wakes, as discussed in the next section.

9.3 Vortex Shedding Sounds

The turbulent wakes of most bodies contain relatively strong vortices which occur in certain geometric configurations and which account for a significant fraction of wake energy. Oscillating forces are produced in connection with the formation of vortex wakes. These forces are responsible for such diverse phenomena as noise in electrical power lines, vibration of radar antennas, fatigue failure of hydraulic turbine blades and collapse of tall smokestacks. They have been studied extensively by mechanical, civil and hydraulic engineers as well as by acousticians and aerodynamicists. In acoustics, oscillating forces on wires produce sounds known as *Aeolian tones*, and similar forces on blades produce components of fan, compressor, pump and propeller noise that are sometimes very strong and are known as *singing*.

Investigations of the subject of vortex shedding phenomena within each of the disciplines mentioned proceeded for many decades almost without interaction between them. Although articles on this subject have appeared for almost 100 years, it was not until the 1960's that Ross (1964) combined results from all the fields to develop a unified hydro-acoustic theory of vortex shedding sounds. The present section is based on that study as revised by more recent experimental and theoretical developments.

Aeolian Tones

As recounted by Richardson (1924), descriptions of *Aeolian harps* in which wind or the draft from a fire produces musical tones are given in a book published in the 17th century. Although the phenomenon had been known since the time of the Greeks, the first scientific investigation was that of Strouhal (1878) who found that frequency of the sound increases with wind speed and decreases with wire diameter. Thus, the dimensionless frequency fD/U_o tends to remain constant. This factor is known as the *Strouhal number*, S_N, and is written

$$S_N \equiv \frac{fb}{U_o} , \tag{9.42}$$

where b is a characteristic transverse dimension of the body. For cylinders, Strouhal found $S_N \doteq 0.185$. Rayleigh (1915) noted that vibrations of the wire occur in a plane perpendicular to the direction of the wind. He associated the motion with the vortex wake that had been observed in water, and also concluded that the Strouhal number must be a function of Reynolds number, finding that Strouhal's experimental results are given by

$$S_N \doteq 0.195 \left(1 - \frac{20}{R_N}\right) . \tag{9.43}$$

Many of the studies of vortex shedding have been concerned with determining the exact relationship between S_N and R_N for rigid cylinders. Figure 9.4 summarizes these results. For very low values of R_N, below about 40, a symmetric vortex pattern is frozen in space. Above this value vortices are shed alternately from one side and the other and the dimensionless frequency is about 0.12, increasing to 0.19 at $R_N = 200$. In the Reynolds number range from 200 to 200,000 the wake is turbulent, but a discrete vortex pattern persists and the Strouhal number is practically constant at $S_N = 0.20 \pm 0.01$. Above about $R_N = 2 \times 10^5$, the situation is unclear. From this value to $R_N \doteq 3 \times 10^6$, the wake is highly turbulent and whatever vortices occur do not seem to have a definite frequency. Relf and Simmons (1925) made wake hot-wire measurements and found a peak in the spectrum with $S_N > 0.4$ in this region. Similar results were reported by Delany and Sorensen (1953), Itaya and Yasuda (1961) and Bearman (1969). However, Fung (1958) and Blyumina and Fedyaevskii (1968) reported finding $S_N \doteq 0.20$ throughout. Above this region of weak discrete vorticity, stronger vortices again occur, and Roshko (1961) found $S_N \doteq 0.27$ for R_N from 3×10^6 to 10^7.

All of the results summarized above are presumably for rigid cylinders in a free environment. Any departures from ideal conditions may cause significant changes of observed Strouhal frequencies.

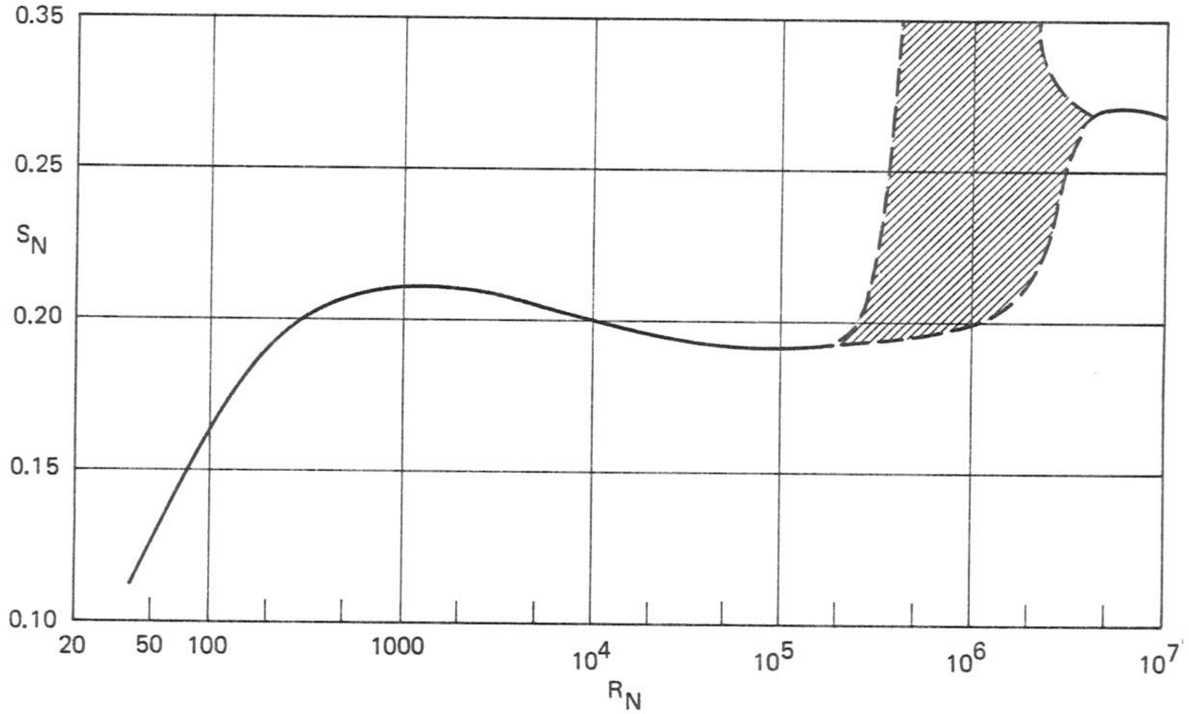

Fig. 9.4. Strouhal-Reynolds Number Relation for Circular Cylinders

Vortex Wakes of Bluff Bodies

Figure 9.5 shows a *vortex street* consisting of two rows of alternating vortices of strength Γ in the wake of an elliptic cylinder of thickness b. The vortex rows are separated by h and the distance between vortices in the same row is a. The vortices produce their own velocity field superimposed on the free stream velocity, U_o. The velocity induced at any vortex by all the other vortices is u_s. Von Karman and Rubach (1912) carried out a stability analysis of an infinite vortex street, finding

$$\frac{h}{a} = 0.28 \ . \tag{9.44}$$

Measurements of the spacing ratio of actual vortex streets are generally in fair agreement with this value, though ratios as high as 0.35 are not uncommon. Birkhoff (1953) has reasoned that, while a

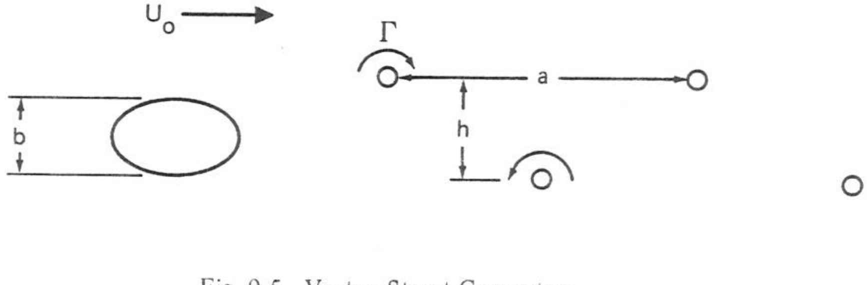

Fig. 9.5. Vortex Street Geometry

remains constant as the wake decays, h increases due to the action of viscosity. Near the body, spacing ratios are usually close to that given by Eq. 9.44.

Von Karman and Rubach computed the induced velocity for an infinite vortex street, finding

$$u_s = \frac{\Gamma}{2a} \tanh \frac{\pi h}{a} \doteq \frac{\Gamma}{2\sqrt{2}\,a}\left[1 + \frac{\pi}{\sqrt{2}}\left(\frac{h}{a} - 0.281\right)\right] \doteq \frac{3\Gamma}{8a}\sqrt{\pi\frac{h}{a}} . \tag{9.45}$$

Assuming the spacing to be close to equilibrium,

$$\frac{u_s}{U_o} \doteq \frac{\Gamma}{2\sqrt{2}aU_o} . \tag{9.46}$$

The Strouhal number for the movement of vortices past a fixed point in the wake is

$$S_N = \frac{U_o - u_s}{a}\frac{b}{U_o} = \left(1 - \frac{u_s}{U_o}\right)\frac{b}{a} \doteq 0.28\left(1 - \frac{u_s}{U_o}\right)\frac{b}{h} , \tag{9.47}$$

showing that both relative wake width and induced velocity affect the observed frequency.

In some Reynolds number ranges, a significant fraction of the drag is associated with wake vortices. Von Karman and Rubach (1912) equated momentum carried downstream by vortices to form drag, F_{D_F}, finding

$$F_{D_F} \doteq \frac{\rho_o \Gamma h}{a}(U_o - 2u_s) + \frac{\rho_o \Gamma^2}{2\pi a} . \tag{9.48}$$

Defining a form drag coefficient similar to Eq. 7.73 and assuming equilibrium spacing, Eq. 9.48 leads to

$$C_{D_F} \equiv \frac{F_{D_F}}{\frac{1}{2}\rho_o U_o^2 b} \doteq 4\sqrt{2}\frac{h}{b}\frac{u_s}{U_o}\left(1 - 0.4\frac{u_s}{U_o}\right) . \tag{9.49}$$

Form drag is proportional to the relative wake width and Strouhal number varies inversely with this quantity. Krzywoblocki (1945) multiplied Eqs. 9.47 and 9.49, getting

$$S_N \cdot C_{D_F} \doteq 1.6\frac{u_s}{U_o}\left[1 - 1.4\frac{u_s}{U_o} + 0.4\left(\frac{u_s}{U_o}\right)^2\right] , \tag{9.50}$$

independent of relative wake width. This relation is plotted in Fig. 9.6. Roshko (1961) noted the inverse behavior of S_N and C_{D_F}. Since the drag coefficient drops precipitously in the critical Reynolds number region around $3 \cdot 10^5$, he found an increase of Strouhal numbers in this region to be quite believable. However, if very little energy were to go into vortex streets in this region, then vortex strengths would be low and one would expect relative induced velocities to be low as

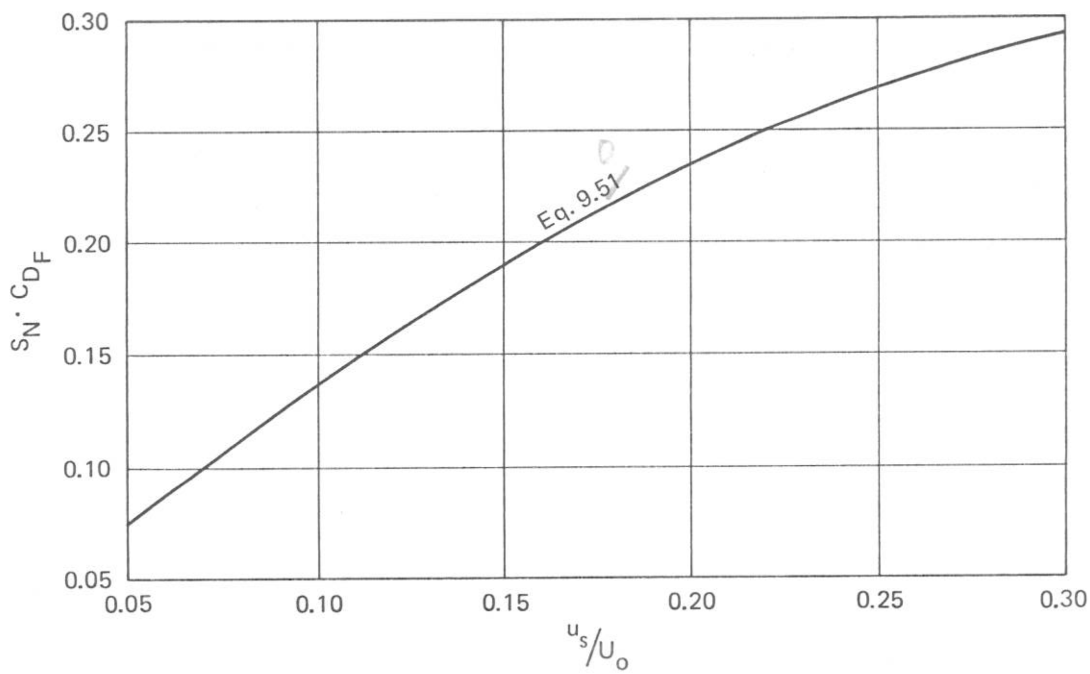

Fig. 9.6. Product of Strouhal Number and Form Drag Coefficient from Eq. 9.50

well. By Eq. 9.49, C_{D_F} would decrease relatively more than S_N would increase and the product would decrease in accordance with Fig. 9.6. Figure 9.7 is a plot of the product of S_N and C_D as a function of Reynolds number for vortex shedding from cylinders. Except in the critical regime, the product is close to constant at 0.22, implying that $u_s \doteq 0.18\ U_o$ and that vortex strength is practically constant outside that region.

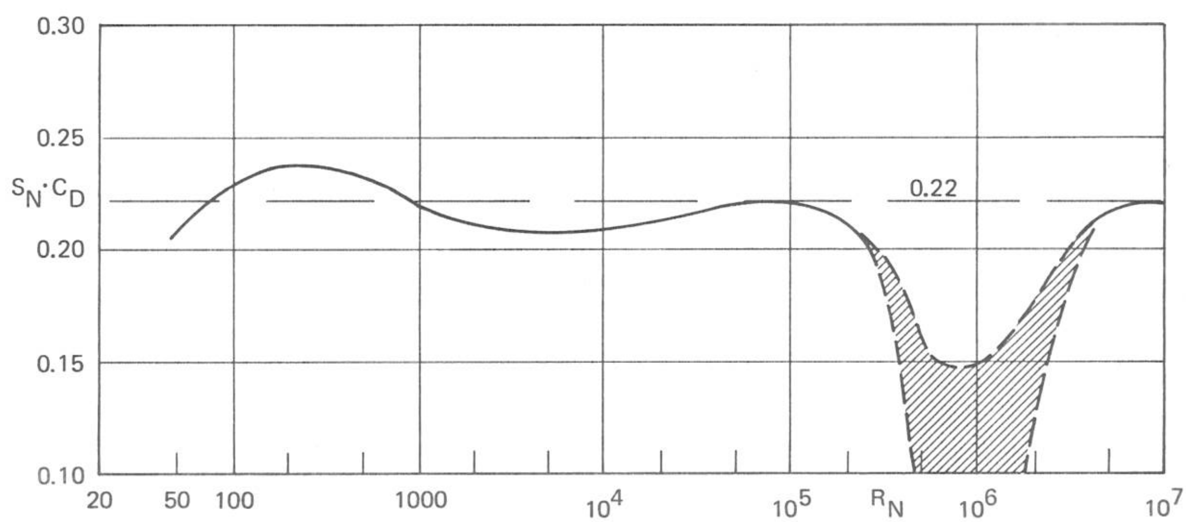

Fig. 9.7. Product of Strouhal Number and Drag Coefficient for Cylinders

These results for cylinders can be generalized to other bluff bodies, since nothing in the analysis was specific to circular or elliptic cylinders. Thus Fage and Johansen (1927, 1928) found that the Strouhal number for flat plates at large angles to a flow is constant at 0.15 over a wide range of angles provided b is the projection of the plate normal to the flow. They also found $u_s \doteq 0.16\ U_o$. Roshko (1955) studied a wide variety of bluff bodies, finding

$$S_R \equiv \frac{fh'}{U_s} \doteq 0.165 \tag{9.51}$$

for a wide range of Reynolds numbers. The reference velocity U_s in this relation is the velocity at the point of separation of the flow from the body and h' is the theoretical distance between the separated free streamlines as determined downstream where they have become parallel. Abernathy (1962) studied separated flows from flat plates at various angles to a flow, finding

$$S_A \equiv \frac{fh''}{U_s} \doteq 0.155 \ , \tag{9.52}$$

where h'' is the measured separation between the centers of the shear layers. Bearman (1967) used separations of the vortex streets and free-streamline velocity, U_s, in defining a Strouhal number,

$$S_B \equiv \frac{fh}{U_s} \ , \tag{9.53}$$

which he found to be 0.18 for a wide variety of wakes. The three modified Strouhal numbers all use the free-streamline velocity, U_s, since this quantity is more fundamental than the flow speed.

Oscillating Forces Associated with Vortex Wakes

Formation of vortices and motion of a vortex street away from the body shedding the vortices induce a time-varying velocity component in the flow field about that body and consequently a time-varying pressure field. The result is an oscillating force component practically perpendicular to the direction of flow, which force is a source of dipole sound as well as of vibrations of the body.

The instantaneous induced velocity field and force resulting from vortex street motion are dominated by the vortex closest to the body. It is therefore useful to calculate these quantities for the motion of a single vortex. Consider a vortex of strength Γ moving with speed $U_o - u_s$ away from a cylinder of diameter D, as shown in Fig. 9.8. As explained by Milne-Thomson (1950), the effects of this vortex can be treated in terms of an image vortex located within the cylinder at radius s_i given by

$$s_i = \frac{(D/2)^2}{s} = \frac{(D/2)^2}{\sqrt{x^2 + \left(\frac{h}{2}\right)^2}} \ . \tag{9.54}$$

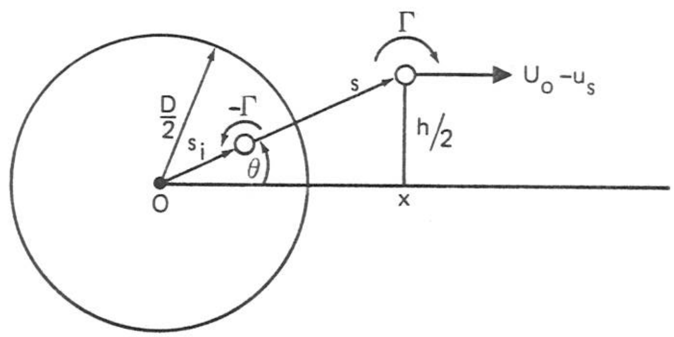

Fig. 9.8. Image Vortex for Vortex Outside a Cylinder

The lift component of the induced force at any instant is given by

$$F_{L_i} = -\rho_o \Gamma \dot{s}_i \cos\theta \ . \tag{9.55}$$

Taking the time derivative of Eq. 9.54, it follows that

$$F_{L_i} = \rho_o \Gamma (U_o - u_s) \frac{x^2 (D/2)^2}{\left[x^2 + \left(\frac{h}{2}\right)^2\right]^2} \ . \tag{9.56}$$

The maximum instantaneous lift that could be experienced would be that for a vortex at $x = D/2$, for which

$$\hat{F}_{L_i} = \frac{\rho_o \Gamma (U_o - u_s)}{\left[1 + \left(\frac{h}{D}\right)^2\right]^2} \ . \tag{9.57}$$

As the first vortex proceeds downstream, a second one of opposite sign is formed at $-h/2$ leading to a negative force equal to that given by Eq. 9.57. The rms oscillating lift for the fundamental component at shedding frequency is estimated to be approximately half of $\hat{F}_{L_i}$. From this fact and from Eq. 9.46 it follows that

$$\tilde{C}_L \doteq 2\sqrt{2} \frac{u_s}{U_o} \left(1 - \frac{u_s}{U_o}\right) \frac{a/D}{\left[1 + \left(\frac{h}{D}\right)^2\right]^2} \ . \tag{9.58}$$

Comparison with Eq. 9.49 shows that the oscillating lift coefficient due to a single vortex is closely related to the form drag coefficient, their ratio being

$$\frac{\widetilde{C}_L}{C_{D_F}} \doteq \frac{\frac{1}{2}\left(\frac{a}{h}\right)}{\left[1 + \left(\frac{h}{D}\right)^2\right]^2} \left(1 - 0.6\,\frac{u_s}{U_o}\right) . \qquad (9.59)$$

For circular cylinders at Reynolds numbers below 10^5, the lateral separation distance h can be taken as approximately equal to the diameter and $u_s/U_o \doteq 0.16$. It follows that the value of the rms oscillating lift coefficient calculated from Eq. 9.58 is about 40% of the form drag, which result is in agreement with an analysis originally published by Ruedy (1935).

Three factors not considered in deriving Eqs. 9.57-9.59 act to reduce the magnitude of the oscillating force. The first is that vortices do not form right at the cylinder, at $x = D/2$, as was assumed. Actually, vorticity is shed at this point into a parallel shear layer. As discussed by Abernathy and Kronauer (1962), such shear layers are unstable and break up into discrete vortices at downstream distances that depend on a number of external factors. Figure 9.9, based on Eq. 9.56, shows the decrease of induced force as a function of the downstream position of vortex formation. Thus, if the vortices were to form at a distance one diameter downstream, for which

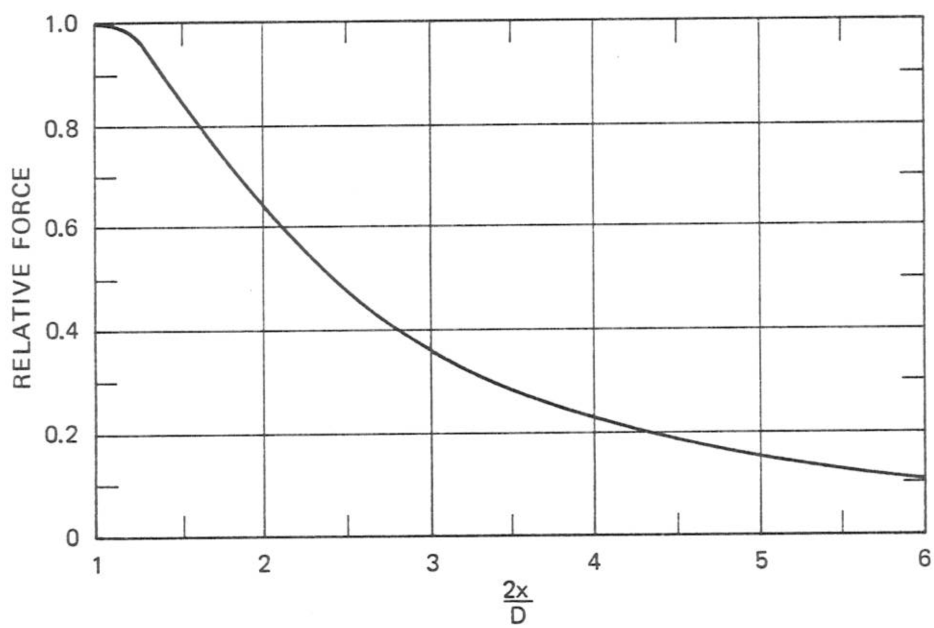

Fig. 9.9. Relative Induced Force as Function of Vortex Position in Fig. 9.8

$2x/D \doteq 3$, the force would be reduced to about one third of that previously calculated. Several investigators have reported that, when tests are run in low-turbulence flows at Reynolds numbers between 200 and 5000 and when care is exerted to prevent cylinder oscillation, formation distances to the first vortex are from 2 to 4 cylinder diameters. The corresponding oscillating force is only 5 to 15% of that for close vortex formation.

A second factor is reduction of the peak force due to induced velocities of all previously shed vortices. This amounts to about 15% and reduces the maximum possible oscillating lift from

40% of the form drag to about one third. Finally, the assumption of two-dimensionality is violated, as will be discussed below.

In view of the sensitivity of oscillating forces to the point of formation of the first vortex and to three-dimensional effects it is not at all surprising that experimental measurements of oscillating lift coefficients show a great deal of scatter. Confirmation of the above analysis is found in the fact that the highest measured values are about half the form drag. Thus, rms lift coefficients as high as 0.5 to 0.6 have been reported by Keefe (1962) and several other investigators. On the other hand, values under 0.1 were measured by Gerrard (1961) and Leehey and Hanson (1970) for Reynolds numbers between 10^3 and 10^4. Ballou (1967) confirmed that distance to vortex formation decreases markedly in the Reynolds number range between 4000 and 6000, corresponding to the observed increase of $\widetilde{C}_L$.

Three-Dimensional Character of Vortex Wakes

The two-dimensional vortex street envisioned by von Karman and Rubach (1912) and other investigators consists of long straight vortices lined up parallel to the axis of the shedding bluff body; this is an idealization not usually found in practice. Flow visualization experiments of Hama (1957), Gerrard (1966), Koopmann (1967) and Berger and Wille (1972) have shown that under some conditions the vortices are shed in a regular manner but at angles of 10 to 30 degrees relative to the cylinder axis, while under other conditions the vortex lines are irregular as well as slanted.

Indications of the three-dimensional character of vortex patterns have also been found in measurements of oscillating pressures and velocities at different positions along a cylinder by Prendergast (1958) and el Baroudi (1960) and in flow pattern measurements by Humphreys (1960). These investigators found phase reversals occurring on the cylinder at lateral distances corresponding to 2 to 5 diameters. Vickery (1966) and Petrie (1974) reported that high levels of free-stream turbulence cause irregularity of vortex patterns for $R_N < 10^4$, for which shedding is normally regular.

Three-dimensionality of the vortex pattern reduces oscillating lift in two ways. Distortion results in lower local lift forces at each element of the cylinder, and phase shifts result in strong cancellation effects when the lift is integrated over the entire length. When slanted vortices are regular, as may occur for Reynolds numbers of 200 to 5000, this cancellation effect can be very strong. When they are irregular due to wake or stream turbulence, force cells each a few diameters long are produced and these add more or less randomly. Thus, Petrie found that in some cases turbulence increases the total oscillating lift. It is apparent that the wide range of measured oscillating lift coefficients that have been reported can be attributed to differences of experimental conditions and resultant differences of the three-dimensional vortex patterns.

Effects of Vibration

Three-dimensional vortex patterns, either regular or irregular, are observed only in the wakes of rigid cylinders. As illustrated by Fig. 9.10, Koopmann (1967) and Griffin et al (1973) reported that the most dramatic effect of slight cylinder vibration is to straighten out the vortices so that near the cylinder they are essentially parallel to the cylinder axis. A second effect of cylinder motion is to assure that vortices form close to the body. Thus, in cases where the oscillating lift is relatively low due to three-dimensional vortex patterns and/or delayed vortex formation, a slight amount of cylinder vibration in synchronism with the vortex shedding may increase the oscillating lift coefficient by a large factor. It seems likely that some freedom to vibrate may have been involved in those experiments that produced exceptionally high values of $\widetilde{C}_L$.

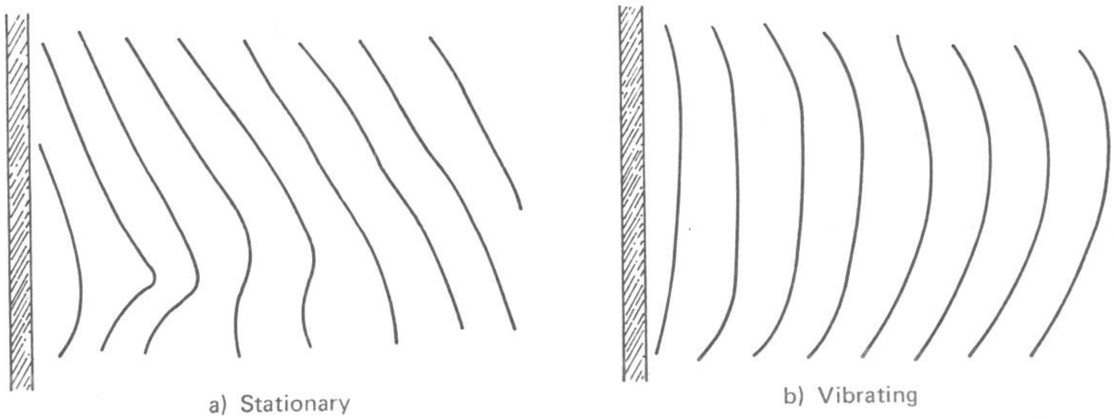

Fig. 9.10. Effect of Cylinder Vibration on Vortex Pattern, after Koopmann (1967)

Oscillations of the body shedding the vortices can also effect the shedding frequency. If a body is driven at a frequency within about ± 10% of the natural shedding frequency, vortices are shed at the cylinder vibration frequency.

The vibratory interaction of the body shedding the vortices with its vortex street explains many of the catastrophic effects of vortex shedding. For example, consider a tall thin structure in a wind. Normally the Stouhal frequency does not coincide with resonance frequencies of the structure; the structure remains rigid, the vortex pattern is irregular and the overall force is low. However, should the shedding frequency approach a resonance, the structure may start to vibrate. This vibration may cause the vortices to straighten, which increases the force and also the vibration up to the point at which the entire vortex pattern is straight and exerts maximum force. In this situation, the vibration amplitude may become so large as to cause structural failure. A similar phenomenon is involved in *singing*, which will be discussed later in this section.

Effects of Sound Fields

Many practical examples of vortex shedding occur in enclosed or partially enclosed spaces. In such cases, interactions of acoustic properties of the spaces with vortex shedding phenomena have been noted by a number of investigators. Parker et al (1966, 1967, 1968 and 1972) found that wind tunnel resonances affect both force amplitudes and frequencies associated with vortex wakes of airfoils. Graham and Maull (1971) and Cumpsty and Whitehead (1971) reported that acoustic resonances cause vortices to shed in a regular manner parallel to the cylinder axis in much the same manner as vibration of the cylinder.

Some investigators have suggested that acoustic feedback mechanisms play a central role in controlling vortex shedding processes in a manner similar to that found in the related phenomenon of edge tones. *Edge tones* are created when a fluid stream impinges on the edge of a flat plate. As analyzed by Richardson (1931), Curle (1953) and Powell (1953, 1961), acoustic feedback from the plate serves to stabilize one of many possible vortex patterns originating in the jet. It is generally agreed that this is not a dominant mechanism for vortex shedding from rigid bodies, but rather that sound fields may play a secondary role much like cylinder vibrations.

Vortex Sounds from Cylinders

Measurements of vortex shedding sounds by Yudin (1942), Gerrard (1955), Richardson (1957) and Etkin et al (1957) reveal a sixth-power dependence of acoustic power on flow speed and a cosine directionality pattern. As discussed in Section 9.1, both of these characteristics are typical of dipole radiation. Yudin (1944) assumed the oscillating force to be proportional to the drag coefficient and derived an expression for the acoustic power which was improved slightly by Blokhintsev (1946).

Most modern analyses are based on an expression derived by Phillips (1956) that includes three-dimensionality effects. Phillips started with Lighthill's equation and derived a general expression similar to Eq. 9.28 for the acoustic intensity at a point distant r from a body experiencing a concentrated oscillating force. Expressing the normal cross-sectional area, S, by the product of the transverse dimension, b, and a spanwise length, ℓ, as shown in Fig. 9.11, his result may be written

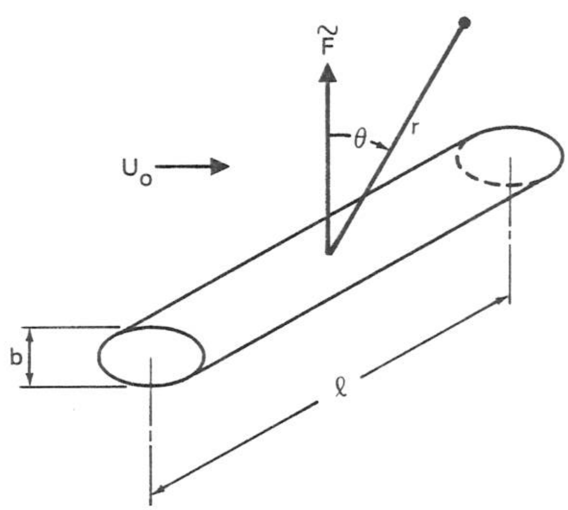

Fig. 9.11. Geometric Arrangement for Acoustic Calculation

$$I(\theta) = \frac{\rho_o U_o^3 \ell^2}{16 r^2} \left(\frac{fb}{U_o}\right)^2 \left(\frac{U_o}{c_o}\right)^3 \tilde{C}_F^2 \cos^2\theta \quad . \tag{9.60}$$

Phillips noted that this expression is only valid over a spanwise distance for which vortex shedding is coherent. For a cylinder of length L he estimated that there would be L/ℓ coherent vortex shedding cells. Assuming that their phase angles are related randomly, Phillips summed the intensities, obtaining

$$I(\theta) = \frac{\rho_o U_o^3 \ell L}{16 r^2} S_N^2 \tilde{C}_F^2 M^3 \cos^2\theta \tag{9.61}$$

for the total sound intensity from a body of length L and

$$W_{ac} = \frac{\pi \rho_o U_o^3 \ell L}{12} S_N^2 \tilde{C}_F^2 M^3 \tag{9.62}$$

for the radiated power. An important result shown by Phillips' equation is the dependence of sound power on the coherence length. Thus, factors such as vibration and acoustic resonances that tend to straighten out the vortex pattern may increase the sound radiated more by increasing the correlation length, ℓ, than by increasing the oscillating lift coefficient, $\widetilde{C}_F$.

Ross (1964) rewrote Eqs. 9.61 and 9.62 by introducing the steady-state form drag coefficient, C_{D_F}, obtaining

$$I(\theta) = \frac{\rho_o U_o^3 S}{16r^2} \left(S_N C_{D_F}\right)^2 \left(\frac{\widetilde{C}_F}{C_{D_F}}\right)^2 \left(\frac{\ell}{b}\right) M^3 \cos^2\theta \tag{9.63}$$

and

$$W_{ac} = \frac{\pi \rho_o U_o^3 S}{12} \left(S_N C_{D_F}\right)^2 \left(\frac{\widetilde{C}_F}{C_{D_F}}\right)^2 \left(\frac{\ell}{b}\right) M^3 . \tag{9.64}$$

Based on the data shown in Fig. 9.7 and the relationship between oscillating lift and form drag previously discussed, Ross set $S_N C_{D_F} \doteq 0.22$ and $\widetilde{C}_F \doteq 1/3\ C_{D_F}$, obtaining

$$I(\theta) \doteq 3.4 \times 10^{-4} \frac{\rho_o U_o^3 S}{r^2} \frac{\ell}{b} M^3 \cos^2\theta \tag{9.65}$$

and

$$W_{ac} \doteq (1.4 \times 10^{-3})\, \rho_o U_o^3 S \frac{\ell}{b} M^3 . \tag{9.66}$$

As previously discussed, $\widetilde{C}_F$ is sometimes much smaller than $1/3\ C_D$, in which case Eqs. 9.65 and 9.66 overestimate the radiated sound. The ratio of ℓ to b may be as low as 2 for a rigid cylinder at moderate Reynolds numbers, or ℓ may almost equal L for a cylinder free to vibrate.

Equation 9.64 can be used to estimate the acoustic conversion efficiency for vortex shedding. The mechanical power given by Eq. 9.30 can be written

$$W_{mech} = \frac{1}{2} \rho_o U_o^3 S C_D , \tag{9.67}$$

from which

$$\eta_{ac} = \frac{\pi}{6} \left(\frac{S_N^2 C_{D_F}^2}{C_D}\right) \left(\frac{\widetilde{C}_F}{C_{D_F}}\right)^2 \frac{\ell}{b} M^3 . \tag{9.68}$$

If form drag dominates, then $C_{D_F} \doteq C_D$ and

$$S_N^2 C_{D_F} \doteq 0.053 \frac{b}{h} \tag{9.69}$$

over a wide range of values of the relative induced velocity. With this result, and setting $\widetilde{C}_F \doteq 1/3\ C_{D_F}$, Eq. 9.68 becomes

$$\eta_{ac} \doteq 3 \times 10^{-3} \frac{\ell}{h} M^3 , \qquad (9.70)$$

showing that in many situations the acoustic efficiency depends only on relative coherence length and Mach number.

Phillips (1956) found that the intensity measurements of Holle (1938) and Gerrard (1955) for Reynolds numbers of 200 to 3×10^4 confirm the sixth-power dependence of intensity on flow speed predicted by Eq. 9.61 and agree as to magnitude with this expression, provided the coherence length, ℓ, is taken to be about 5 to 8 diameters. More recently, Leehey and Hanson (1970), measuring vortex-shedding sound from a wire in a low-noise, low-turbulence wind tunnel, found that intensity increases by about the ninth power of flow speed in the Reynolds number range between 3000 and 9000, for which they also found a dramatic increase of the fluctuating lift coefficient associated with changes of the vortex formation distance. A similar result has been reported by Rimsky-Korsakov (1975) based on results of noise measurements from rotating rods.

Sounds from Rotating Rods

The vortex shedding frequency of each section of a rod rotating about its center is a function of radius of that section, and each section of about one diameter in length radiates independently. It follows that

$$I(\theta) \doteq \frac{4.8 \times 10^{-5}}{r^2} \rho_o U_t^3 b D M_t^3 \cos^2\theta \qquad (9.71)$$

and

$$W_{ac} \doteq (2.0 \times 10^{-4})\, \rho_o U_t^3 D b M_t^3 , \qquad (9.72)$$

and that

$$\eta_{ac} \doteq (1.2 \times 10^{-3})\, M_t^3 , \qquad (9.73)$$

where subscript t refers to values computed at the tip. These equations apply to rods formed of bluff sections, such as cylinders. They must be modified when dealing with airfoil sections, as will be discussed next. Also, as noted by Ross (1964), they appear to overestimate the sound measured by Stowell and Deming (1936) and by Yudin (1944) by from 3 to 18 dB. Rimsky-Korsakov (1975) reported agreement with these results for rods having tip Reynolds numbers between 2×10^4 and 4×10^5, levels at smaller Reynolds numbers being as much as 15 dB lower.

Vortex Wakes of Airfoils

Turbulent wakes of airfoils at low angles of attack contain discrete vortices that resemble von Karman vortex streets of bluff bodies. Unlike bluff bodies, however, no simple relation exists between body dimensions and shedding frequency. The reason for this is that wake widths depend on both development of the boundary layer over the section and detailed trailing edge geometry.

Thus, Tyler (1928) and Lehnert (1937), defining Strouhal number in terms of airfoil thickness, found measured values to be strong functions of Reynolds number. Taneda (1958) found the variation to be as $R_N^{1/2}$ for flat plates having negligible trailing edge thicknesses. Gongwer (1952) recognized that the appropriate dimension should be the separation distance of the vortex sheets. He added the boundary-layer momentum thickness, θ, to the trailing edge thickness, e, finding

$$\frac{f(e + \theta)}{U_o} \doteq 0.185 \; . \tag{9.74}$$

The effect of trailing edge shape on the strength of the oscillating lift associated with vortex wakes has been studied by a number of groups in connection with design of hydraulic turbine and propeller blades for minimum noise. Three factors ~~that~~ apparently influence the force: width of the street determines strength of the individual vortices and shedding frequency, in accordance with Eqs. 9.45 and 9.75; distance of point of formation of vortices from the trailing edge strongly affects force amplitude; and sharpness of the trailing edge controls its susceptibility to vibration, which vibration would be expected to increase the oscillating force by the mechanism previously discussed. Figure 9.12 summarizes results published by Donaldson (1956), Ippen et al (1960),

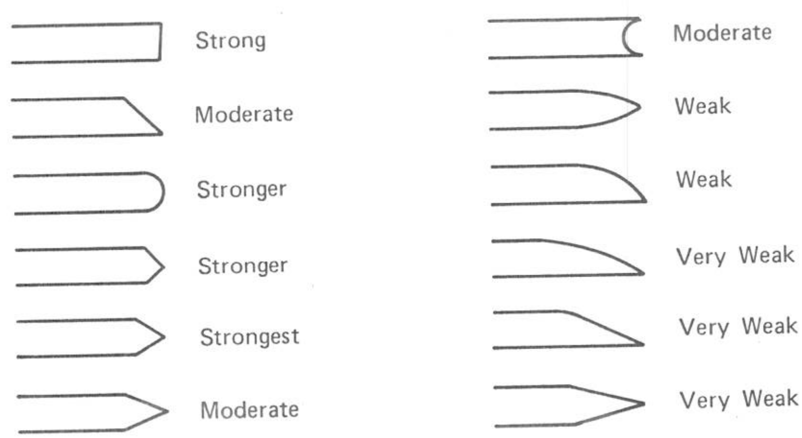

Fig. 9.12. Evaluation of Trailing Edges from Flat Plate Oscillating Force Measurements

Heckestad and Olberts (1960) and Toebes and Eagleson (1961) which confirm our expectation that an edge with strong vortices formed close to the tip experiences maximum force, and an edge which produces a narrow vortex wake is most desirable.

The peak oscillating pressure produced at the trailing edge of an airfoil has been shown by Blake (1975) to occur at the point where the streamline representing the center of the vortex street leaves the airfoil, as assumed by von Karman and Sears (1938). In many cases this is right at the tail, but it can be somewhat upstream. It follows that peak sound is radiated from near the trailing edge and not from the wake.

Vortex Sounds from Rotating Blades

There have been no conclusive experiments in which airfoil vortex sounds were measured simultaneously with appropriate hydrodynamical characteristics of the wake. Ross (1964) suggested that Eqs. 9.65, 9.66 and 9.70-9.73, which were derived for bluff bodies, might apply unchanged to more streamlined sections. However, there is reason to believe that only a small fraction of the wake energy of airfoils occurs as discrete vortices and that the induced velocity, u_s, is therefore a smaller fraction of the flow speed. If this is the case, then the product of S_N and C_{D_F} should be somewhat smaller than for cylinders. Also, vortices may form relatively far downstream, thereby reducing the ratio of $\widetilde{C}_L$ to C_{D_F}. For these reasons, the author believes that equations developed for bluff sections overestimate noise levels for airfoils and for rotating blades having airfoil sections by about 10 dB.

9.4 Noise from Fans and Blowers

Noise Mechanisms

Noise spectra of fans and blowers are almost entirely produced by the oscillating-force mechanisms discussed in the previous two sections. Spectra of fans include both tonal and broadband components. Tonal components occur at multiples of blade passage frequency and are caused by flow assymmetries and rotor-stator interactions. Sources of broadband noise include wake vortex shedding, boundary-layer turbulence and blade operation in turbulent inflows. While there is good agreement throughout the literature on causes of tonal radiation, considerable controversy exists as to whether vortex shedding or boundary-layer turbulence is the dominant source of broadband noise when rotors operate under non-turbulent inflow conditions. Thus, Wells and Madison (1957) and Rimsky-Korsakov (1975) mentioned only vortex shedding, and Mugridge and Morfey (1972, 1973) discussed only boundary-layer turbulence. Sharland (1964) calculated levels from both sources and concluded that noise from vortex shedding is from 3 to 10 dB stronger than noise from boundary-layer turbulences. He also demonstrated that under turbulent inflow conditions blade interaction with incoming turbulence is the dominant broadband mechanism. Barry and Moore (1971) have noted that high shaft-rate harmonics caused by blade-to-blade variations also contribute to high-frequency broadband fan spectra.

Spectra

The relative importance of tonal and broadband spectra versus overall spectrum shape depends strongly on the type of fan or blower. Fans are divided into two categories: axial and centrifugal. *Axial-flow fans*, often referred to as *propeller fans*, may operate against little or no static pressure, may be in housings or may be free standing. *Centrifugal fans and blowers* impart a radial motion to the fluid and operate against a significant static pressure drop. They operate at lower tip speeds and usually have more blades than axial-flow units. (Rotary positive displacement blowers, used to deliver small volumes of gas against high pressures, are not classed as fans.)

Blade passage frequencies of axial flow and centrifugal machines are generally in the same range, since axial machines have fewer blades but operate at higher rpm's. Typically, the fundamental blade frequency lies between 100 and 350 Hz. Some centrifugal blowers operate at speeds as high as 12,000 rpm and have blade tonals above 2 kHz. Blade tonals are usually the strongest components of the spectrum, being only about 4 to 8 dB lower than the overall level. Above the blade frequency, the spectra from centrifugal machines decrease quite markedly while those of axial-flow units are relatively flat. Figure 9.13 compares average octave-band spectra of the two types of fans. Exceptions to these spectra occur when excitation frequencies coincide with duct resonances, in which case strong peaks occur well above the blade fundamental.

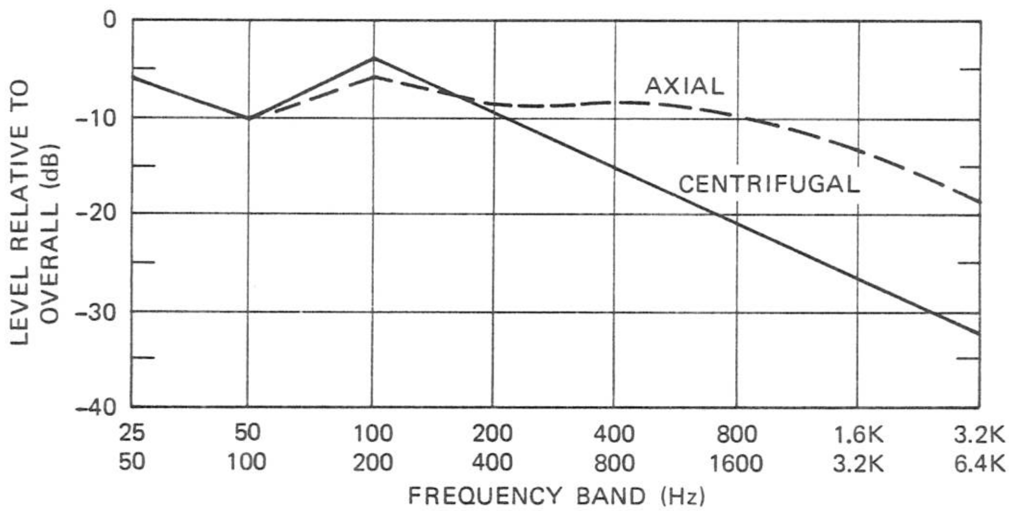

Fig. 9.13. Average Relative Spectra of Axial and Centrifugal Fans

Noise Levels

Intensities from all of the fluctuating-force noise mechanisms discussed in the previous two sections were shown to vary as the sixth power of speed. Yet measurements of fan noise by Peistrup and Wesler (1953), Goldman and Maling (1959) and Sharland (1964) all show weaker dependence on speed, fifth power being typical. There are two major reasons for the discrepancy. First, blade tonals depend on inflow assymmetries caused by upstream stators and other structures. Such wakes tend to become less severe with increasing Reynolds number, thus causing $\widetilde{C}_L$ to decrease slightly with increasing speed. Secondly, the third-power dependence on Mach number found in Eq. 9.28 is valid only when acoustic wavelengths are large compared to the body experiencing the oscillating force, i.e., $kD < 1$. Lowson (1970) has shown that for $kD > 2$ the expressions previously derived for noise from fluctuating-force sources should be divided by $2\,kD$, where

$$kD = \frac{2\pi mBnD}{c_o} = 2mBM_t \quad . \tag{9.75}$$

It follows that at high frequencies and/or high tip speeds fan noise attributable to fluctuating forces should vary at a rate no greater than the fifth power of rotational speed.

Empirical scaling formulas given by Goldman and Maling (1955), Wells and Madison (1957) and Allen (1957) all agree on the fifth-power dependence of noise of a given fan on rpm. Taking into account the approximate cubic dependence of mechanical power on speed, it follows that the acoustic efficiency of a given fan varies as the square of its rpm. However, as noted by Beranek et al (1955), when fans operate near full speed, the acoustic efficiency is nearly constant. Within about ± 5 dB, they found

$$\eta_{ac} \doteq 1.5 \times 10^{-6} \tag{9.76}$$

for a number of *centrifugal fans* rated between 1 and 50 hp. Peistrup and Wesler found the acoustic efficiencies of *axial-flow fans* to be somewhat higher, being given by

$$\eta_{ac} \doteq 10^{-5} \quad . \tag{9.77}$$

The difference is primarily attributable to higher operating tip Mach numbers of axial flow units.

Equations 9.76 and 9.77 give average values of acoustic conversion efficiencies for the two major classes of fans. As noted, fans with higher than normal tip speeds will be noisier and those with lower tip speeds will generally be quieter than the average. The importance of tip speed in controlling fan noise was recognized by Zinchenko (1957) who presented a formula for noise levels of centrifugal blowers as a function only of tip speed. This can be written

$$PWL \doteq 115 + 55 \log \frac{U_t}{100\ m/s} \quad , \tag{9.78}$$

where PWL is acoustic power level in dB relative to 10^{-12} W.

A relation for centrifugal fan noise as a function only of horsepower was given by Beranek et al (1955) as

$$PWL \doteq 90 + 10 \log hp \quad . \tag{9.79}$$

Allen (1957) found that fans make more noise when they operate against a higher static pressure, and Heidmann and Feiler (1974) found a strong correlation with temperature rise of the gas being moved. Thus, Allen suggested

$$PWL \doteq 86 + 10 \log hp + 10 \log \Delta p \tag{9.80}$$

for centrifugal fans, where Δp is measured in cm of water, and Heidmann and Feiler's results can be expressed by

$$PWL \doteq 91 + 10 \log hp + 10 \log \Delta\theta \quad , \tag{9.81}$$

where $\Delta\theta$ is temperature rise in degrees Centigrade. Other factors that affect fan noise include rotor-stator spacing and blade skew of axial fans and impeller clearance and blade slope of centrifugal fans, as discussed by Neise (1976).

Positive Displacement Blowers

Rotary positive displacement blowers of the Roots type are used extensively as scavenging blowers to supply air to two-stroke-cycle diesel engines. They consist of two, three or four intermeshing rotors that force air through a semicircular casing. In many instances they are the predominant source of engine room noise, as reported by Zinchenko (1957). Spectra from these units are dominated by harmonics of the rotor meshing frequency, given by twice the product of the rotational speed and the number of rotors. This may be as low as 30 Hz or as high as 400 Hz. While the fundamental is usually the strongest component, Priede (1966) found that each blower has bands of frequencies for which higher harmonics are enhanced; these bands appear to be related to cavity resonances of the unit. Priede also concluded that rotor tip speed and number of rotors are controlling parameters.

Rotary blowers are extremely noisy. Power levels of over 120 dB re 10^{-12} W are common and even levels as high as 140 dB are sometimes experienced. The noise is especially unpleasant since it is dominated by tonal components. Zinchenko suggested the use of large, low-speed units in order to lower the dominant frequencies and thereby reduce annoyance. He also proposed the use of multiple reflection intake mufflers to achieve at least 25 dB of noise reduction above 500 Hz.

9.5 Propeller Singing

Marine propellers sometimes emit strong tones between 100 and 1000 Hz. Known as *propeller singing*, this phenomenon has been recognized for about 50 years. Similar singing phenomena have also been observed in hydraulic turbines. The sound is sometimes so intense as to be very annoying, and blade vibrations associated with it are often strong enough to produce fatigue failure. For these reasons, early efforts by engineers were primarily aimed at eliminating the problem when it occurred, and more recent scientific investigations have been motivated by the desire to design blades that avoid singing altogether.

A notable characteristic of singing is its dependence on otherwise unimportant features. It is not uncommon for one propeller of a set of seemingly identical propellers to sing while others in the set do not. Most often only one blade of a propeller actually sings, and this occurs only during part of its revolution. Occasionally two blades sing, but at somewhat different frequencies. Since small physical differences determine whether or not a blade will sing, it is not surprising that the literature contains many apparently conflicting cures for this problem. Thus, sharpening of leading edges, sharpening of trailing edges and blunting of trailing edges have all been reported to eliminate singing in specific instances.

Early papers of Gutsche (1937), Hunter (1937), Kerr et al (1940) and Hughes (1945) attributed singing to a wide variety of possible causes, including cavitation, hammer-like blows of wake variations, stalling, shaft-bearing friction, and vortex shedding. Work (1951) described singing as "vibration of propeller blades excited by hydrodynamic forces." Gongwer (1952), Gutsche (1957) and Krivtsov and Pernik (1957) all confirmed the dominance of vortex shedding and it is now recognized that vortex shedding causes most cases of blade singing. Thus, Burrill (1946-1949) and Hughes (1949) found coincidence of singing frequencies with resonance frequencies of blade vibrations; Lankester and Wallace (1955) found that propeller sounds emitted in the absence of singing have a broad peak about an octave wide in the same region of the frequency spectrum where singing occurs; and Van de Voorde (1960) correlated singing susceptibility with trailing edge shape.

Current understanding of blade singing is based on the description of vortex shedding phenomena given in a previous section. Under normal operating conditions, each section of a propeller blade has a vortex wake, frequencies of which differ from section to section because of radial variations of both relative flow speed and trailing edge thickness. The sound emitted therefore covers a bandwidth of about a half octave to an octave and the intensity is within 10 dB of that given by Eq. 9.71. Each blade also has a large number of resonant vibrational frequencies, a few of which involve in-phase vibration of at least a quarter of the blade trailing edge. It is these vibrational modes that are excited most easily by forces applied along the trailing edge. If one of these easy-to-excite vibrational modes lies within the band of vortex shedding frequencies, then the trailing edge may start to vibrate. Vortices in the immediate vicinity would then also shed at this frequency, increasing the coherence length and consequently increasing both force and vibrational amplitude. The process would continue to build up until a large part of the blade would be participating and at this point certain non-linear effects would limit the motion. Ross (1964) estimated the intensity from singing by assuming that about 25% of a blade participates in the radiation, finding

$$I(\theta) \doteq \frac{3 \times 10^{-6}}{r^2} \rho_o U_t^3 D^2 M_t^3 \cos^2 \theta \ . \tag{9.82}$$

Figure 9.14 shows the vortex pattern of a blade during singing, the vortices having been made visible by cavitation.

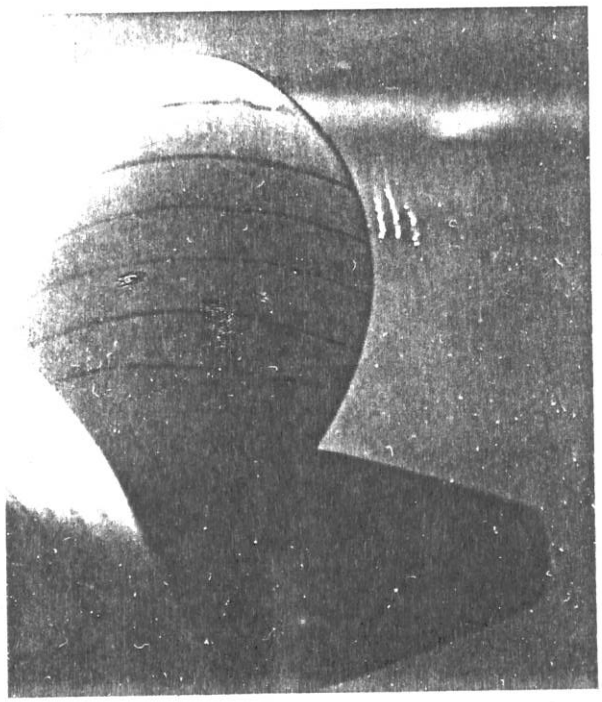

Fig. 9.14. Singing Propeller, as Photographed in Water Tunnel at Hamburg Shipbuilding Research Establishment

With this understanding of singing it is now clear why singing is such a critical phenomenon. Only a few vibrational modes of a blade can be readily excited by trailing edge excitation. One of these must coincide with a vortex shedding frequency. Any change of a blade that either changes natural frequencies or vortex shedding frequencies will probably eliminate singing. In this connection, it should be noted that blades with relatively straight trailing edges are more prone to singing than those with curved edges.

In a study of singing carried out in a water tunnel, Cumming (1965) found that the range of operating conditions over which singing is encountered is smaller than that for which singing can be maintained once it has started. He also confirmed that only a small fraction of blade vibrational modes is likely ever to be involved.

Apparently singing requires appreciable vibrational amplitudes. Therefore, one way to avoid singing is to reduce the resonant response by building blades of a high damping alloy or incorporating vibration damping treatments of the type described in Section 5.9. Arnold et al (1961) and Eagleson et al (1964) have developed theories for singing that include non-linear effects and demonstrate the importance of damping. Hughes (1949) noted that cavitation bubbles on a blade act to increase the damping by absorbing vibrational energy. This explains the rather common observation that singing ceases when cavitation becomes pronounced.

While very annoying when it occurs, singing is readily curable. Anti-singing trailing edges such as those described by Van de Voorde (1960) and Eagleson et al (1964) can be used and/or blades can be damped.

9.6 Flow-Excited Cavity Resonances

Another example of vortex sound that can attain very high levels and also cause fatigue failure is that of *flow-excited cavity resonances*. As discussed in Section 4.9, when certain conditions are met, vortices shed by flow past a cavity mouth may excite resonances of the cavity which act to further strengthen the vortex pattern and thereby produce intense pressures. The phenomenon is similar to singing in that a vortex motion is strengthened by a vibratory motion. Instead of a solid body the vibration is provided by a confined fluid, and instead of being relatively rare the phenomenon is quite common. An example is sound made by blowing across the mouth of a coke bottle.

Relations governing the frequency of vortex formation for rectangular cutouts are less straightforward than for cylinders. One complicating factor is that several vortices may exist in the mouth of a cavity. Another is that thickness of the boundary layer plays an important role in frequency of vortex formation. Figure 9.15 shows a rectangular cutout of length L and depth h, showing that

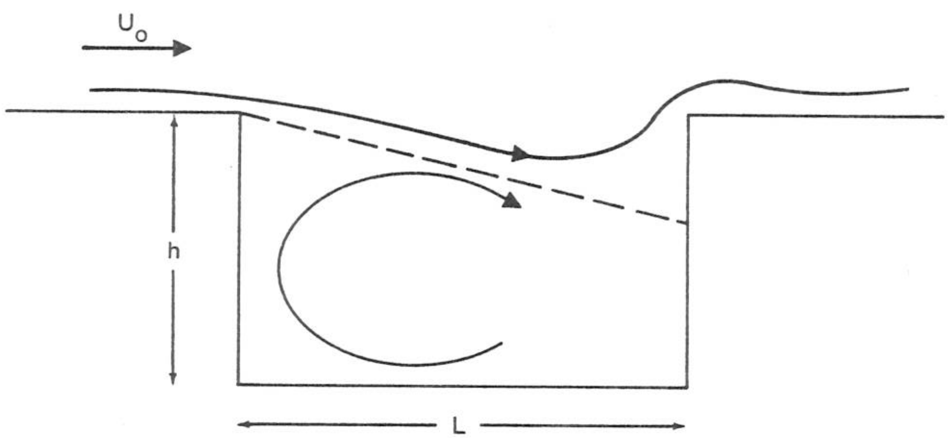

Fig. 9.15. Flow Past a Rectangular Cutout

a shear layer is produced between the outside and inside flows. This shear layer is unstable. According to Dunham (1962) and East (1966), the Strouhal number for vortex development obeys a relation of the form

$$S_N \equiv \frac{fL}{U_o} \doteq m \frac{\bar{u}_v}{U_o} \frac{L}{L - L_v} , \tag{9.83}$$

where m is the number of simultaneous vortices, $\overline{u}_v$ is the average travel speed of a vortex in the shear layer, and L_v is the formation distance of the first vortex. S_N values of 0.3 to 0.6 are common.

East has shown that strong acoustic coupling exists when the frequency also satisfies the relation

$$\frac{fh}{c_o} \doteq \frac{0.25}{1 + \frac{2}{3}\frac{L}{h}} \quad . \tag{9.84}$$

This may readily occur for flow of air past a cavity. In water, however, acoustic coupling can only be a factor if the cavity is very deep or m very large. In water, coupling usually occurs with flexural resonances of the cavity walls, as explained by Dunham and discussed in Section 4.9.

Ingard and Dean (1958) measured intensity of sounds emitted by cavity resonators excited by vortex flows. They found that the sound at resonance increases as U_o^5. This compares with U_o^2 calculated by Blokhintsev (1945). However, Blokhintsev assumed very low level interaction of cavity resonances with vortex formation processes, as would occur at very low speeds, while the Ingard and Dean results are for strong coupling.

Several instances of very strong vortex-excited cavity resonances have been reported, the pressures in some cases being sufficient to cause fatigue cracking of tank walls. Cures are relatively simple, changing the shape of the mouth being the most obvious. If the opening is large enough, vanes can be used to break up the flow. Vortex generators similar to flow spoilers on aircraft wings can also be used to change the nature of the turbulent flow approaching the opening.

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