CHAPTER 3

ACOUSTIC RADIATION FUNDAMENTALS

3.1 General Characterization of Noise Sources

Sound is generated in a fluid medium by any process that causes a non-steady pressure field to occur in that medium. Physical processes that can cause unsteady pressures include the pulsation or vibration of a boundary surface of the medium, the action of a non-steady force on the fluid, turbulent motions in the fluid, and oscillatory temperatures.* Each noise source can be characterized according to its dominant mechanism.

Monopoles, Dipoles and Quadrupoles

Each basic physical mechanism that generates acoustic pressure fields corresponds mathematically to a dominant order of multipole. Thus, volume or mass fluctuations give rise to dominant simple sources, i.e., to zero-order poles called *monopoles*. Examples are pulsating bubbles, pistons in baffles and cavitation. Monopoles are essentially omnidirectional, although directional radiation patterns can be generated by forming arrays of monopoles. Fluctuating forces and vibratory motions of unbaffled rigid bodies are associated with *dipoles* and have cosine directional patterns. Turbulent fluid motions involve distortion without net volume changes or net forces and radiate as *quadrupoles*. Monopoles and dipoles occur only at fluid boundaries, but it is now recognized that quadrupoles can occur within the fluid itself, away from fluid boundaries, in regions of free turbulence where they are associated with fluctuating turbulent shear stresses. Figure 3.1 summarizes the basic physical characteristics of the three lowest order multipole sources.

Radiation Impedance

If a source were a perfectly efficient radiator of sound, the entire motion would be converted into a radiating pressure field. Actual sources create a hydrodynamic non-radiating field as well as an acoustic field. Local pressures associated with the hydrodynamic motion are 90° out of phase with the acoustic component. These concepts are embodied in the radiation impedance

$$\underline{Z}_r = R_r + iX_r \, , \tag{3.1}$$

so defined that the resistance is proportional to the acoustic power and the reactance measures the sloshing hydrodynamic motion. Usually one can find a mean velocity associated with a noise source, in which case acoustic resistance is related to acoustic power by

*Thermal sources of sound are not generally considered when dealing with noise in liquids, although recent experiments with laser beams in water have produced sound by this mechanism.

$$R_r = \frac{W_{ac}}{\overline{U^2}} . \quad (3.2)$$

Radiation reactance is related to hydrodynamic reactive power by a similar relationship.

Radiation impedance measures the reaction of the medium on a source. As such, it is generally proportional to the impedance of the medium, $\rho_o c_o$, and to the area of the source, S_o. The *specific radiation resistance* is the non-dimensional normalized form of the radiation resistance defined by

$$\sigma_r \equiv \frac{R_r}{\rho_o c_o S_o} = \frac{W_{ac}}{\rho_o c_o S_o \overline{U^2}} , \quad (3.3)$$

and the *specific radiation reactance*, σ_x, is the corresponding non-dimensional form of the radiation reactance.

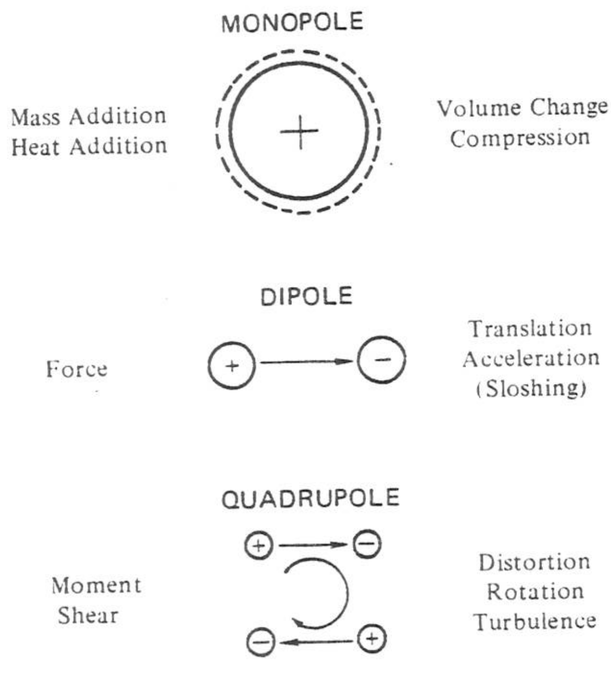

Fig. 3.1. Types of Sound Sources

Radiation Efficiency

The *radiation efficiency* introduced in Chapter 1 by Eq. 1.4 is the ratio of acoustic power to the total power involved in the acoustic and hydrodynamic fluid motions. Thus,

$$\eta_{rad} \equiv \frac{W_{ac}}{W_{ac} + W_{slosh}} = \frac{R_r}{|Z_r|} = \frac{R_r}{\sqrt{R_r^2 + X_r^2}} = \frac{\sigma_r}{\sqrt{\sigma_r^2 + \sigma_x^2}} . \quad (3.4)$$

Some authors prefer to call this the *radiation loss factor*, using the term *radiation efficiency* for the non-dimensional radiation resistance defined by Eq. 3.3.

The radiation efficiency of a multipole is dependent on the order of the pole and the ratio of the size of the radiator to a wavelength. For radiators which are small compared to the wavelength

$$\eta_{rad} \sim \left(\frac{a}{\lambda}\right)^{2m+1} \sim (ka)^{2m+1} , \tag{3.5}$$

where m is the order of the pole, being zero for a monopole, one for a dipole and two for a quadrupole. Since many sources in liquids are characterized by having ka small compared to unity, it follows that the lower the order of the source the more efficient it is as an acoustic radiator. When monopoles exist, they generally dominate. Lacking monopoles, dipoles are most important. It will be shown later in this chapter that quadrupole radiation is seldom significant in liquids.

3.2 General Equation for Sound Generation

The roles of fluctuating mass and force as source terms in acoustics were understood by Stokes and Rayleigh in the 19th century, but it was not until the middle of the 20th century that Lighthill (1952) recognized that turbulent shear stresses could also act as sources of sound. Lighthill realized that sounds from jet aircraft could not be explained in terms of simple mass or force sources and looked to the fluctuating fluid flow as the source of this sound. The derivation of Lighthill's differential equation is more straightforward than that given in Section 2.2 for the usual wave equation in that some of the assumptions made in deriving the wave equation are not needed. Lighthill's equation can be derived simply by combining the continuity and momentum equations without assumptions concerning linearity or steadiness or irrotationality of the fluid flow. The resultant equation can be interpreted as a wave equation with source terms. The same procedure can be used to derive a general equation for sound generation by volume and force as well as shear-stress sources.

Derivation

In deriving a general differential equation for sound generation we start with the continuity and momentum equations of fluid mechanics written for regions that include mass and force source terms. These are then combined to form a single equation prior to making the acoustic assumption. Upon making the acoustic assumption, and after some manipulation, a single differential equation is obtained which has the form of a wave equation on the left, but with a number of terms on the right which can be interpreted as source terms. In carrying out this derivation, it is convenient to use the double-subscript tensor notation described in Section 1.5. In this notation, the continuity equation, Eq. 2.32, in a region containing sources is

$$\frac{D\rho}{Dt} + \rho\,(\nabla \cdot \vec{v}) = \frac{\partial \rho}{\partial t} + \frac{\partial(\rho v_i)}{\partial x_i} = q , \tag{3.6}$$

where q is the rate at which new mass is created per unit volume. This equation differs from Eqs. 2.27 and 2.32 only in the addition of the source term.

It is the momentum equation that takes on a different form in a region containing sources. Eq. 2.40 for the rate-of-change of momentum may be written

$$\frac{DM_i}{Dt} = \int_V \left(\frac{D(\rho \mathrm{v}_i)}{Dt} + \rho \mathrm{v}_i \frac{\partial \mathrm{v}_j}{\partial x_j} \right) dV = \int_V \left(\frac{\partial(\rho \mathrm{v}_i)}{\partial t} + \mathrm{v}_j \frac{\partial(\rho \mathrm{v}_i)}{\partial x_j} + \rho \mathrm{v}_i \frac{\partial \mathrm{v}_j}{\partial x_j} \right) dV . \tag{3.7}$$

The second and third terms can readily be combined into a single term. Equating the time-rate-of-change of momentum to the sum of the forces, Eq. 2.43 is replaced by

$$\frac{\partial(\rho \mathrm{v}_i)}{\partial t} + \frac{\partial(\rho \mathrm{v}_i \mathrm{v}_j)}{\partial x_j} = - \frac{\partial p}{\partial x_i} - \rho g \frac{\partial z}{\partial x_i} + f_i , \tag{3.8}$$

where f_i represents the net force per unit volume exerted by any external mechanical forces that may be acting on the fluid. In this form, viscous stresses are not included. Lighthill chose to include viscous stresses, even though they are negligible in all practical calculations of fluid-dynamic noise. He replaced the pressure, p, by a stress tensor, p_{ij}, which includes both the normal stresses and the viscous shear stresses. In tensor notation, the complete momentum equation may be written

$$\frac{\partial(\rho \mathrm{v}_i)}{\partial t} = - \frac{\partial p_{ij}}{\partial x_j} - \rho g_i - \frac{\partial(\rho \mathrm{v}_i \mathrm{v}_j)}{\partial x_j} + f_i , \tag{3.9}$$

where $g_i = -g \; grad \; z$.

A single second-order differential equation can now be derived by taking the partial derivative of Eq. 3.6 with respect to time,

$$\frac{\partial^2 \rho}{\partial t^2} + \frac{\partial^2(\rho \mathrm{v}_i)}{\partial t \partial x_i} = \frac{\partial q}{\partial t} , \tag{3.10}$$

and the divergence of Eq. 3.9,

$$\frac{\partial^2(\rho \mathrm{v}_i)}{\partial x_i \partial t} = - \frac{\partial^2 p_{ij}}{\partial x_i \partial x_j} + \frac{\partial(\rho g_i)}{\partial x_i} + \frac{\partial f_i}{\partial x_i} - \frac{\partial^2(\rho \mathrm{v}_i \mathrm{v}_j)}{\partial x_i \partial x_j} . \tag{3.11}$$

Subtracting

$$c_o^2 \frac{\partial^2 \rho}{\partial x_i^2} = \delta_{ij} c_o^2 \frac{\partial^2 \rho}{\partial x_i \partial x_j} = \frac{\partial^2(c_o^2 \rho \delta_{ij})}{\partial x_i \partial x_j} \tag{3.12}$$

from both sides of Eq. 3.10 and then combining with Eq. 3.11, one obtains

$$\frac{\partial^2 \rho}{\partial t^2} - c_o^2 \frac{\partial^2 \rho}{\partial x_i^2} = \frac{\partial q}{\partial t} - \frac{\partial(\rho g_i)}{\partial x_i} - \frac{\partial f_i}{\partial x_i} + \frac{\partial^2(p_{ij} + \rho \mathrm{v}_i \mathrm{v}_j - c_o^2 \rho \delta_{ij})}{\partial x_i \partial x_j} . \tag{3.13}$$

Lighthill recognized that the last term of Eq. 3.13 represents several types of stresses. He combined these stresses into a single stress tensor, writing

$$\tau_{ij} \equiv \rho v_i v_j + p_{ij} - c_o^2 \rho \delta_{ij} \,. \tag{3.14}$$

Since the gravitational force is conservative, its divergence is zero, and

$$\frac{\partial^2 \rho}{\partial t^2} - c_o^2 \frac{\partial^2 \rho}{\partial x_i^2} = \frac{\partial q}{\partial t} - g_i \frac{\partial \rho}{\partial x_i} - \frac{\partial f_i}{\partial x_i} + \frac{\partial^2 \tau_{ij}}{\partial x_i \partial x_j} \,. \tag{3.15}$$

Equation 3.15 applies to instantaneous values of the physical quantities, which are the sums of the steady-state values and fluctuating components. Making the acoustic assumption for each quantity, as in Eq. 2.19, subtracting the equation that applies when there are no fluctuating components, and neglecting a residual gravitational term, one obtains a differential equation for the fluctuating components:

$$\frac{\partial^2 \rho'}{\partial t^2} - c_o^2 \frac{\partial^2 \rho'}{\partial x_i^2} = \frac{\partial q'}{\partial t} - \frac{\partial f_i'}{\partial x_i} + \frac{\partial^2 \tau_{ij}}{\partial x_i \partial x_j} \,. \tag{3.16}$$

This generalization of Lighthill's equation includes mass flux and force sources as well as the stresses which he originally considered.

As discussed in Chapter 1, underwater acoustics usually deals with acoustic pressures rather than with fluctuating densities. Equation 3.16 can be transformed into a similar equation for acoustic pressure only by making assumptions that the stress-strain relationship is linear and that spatial variations of ambient quantities are negligible (assumptions 8 and 11 in Section 2.2). Thus, using Eq. 2.22 to relate the acoustic pressure to the fluctuating density and assuming ∇a to be negligible,

$$\nabla^2 p' - \frac{1}{c_o^2} \ddot{p}' = - \dot{q}' + \nabla \cdot \vec{f}' - \frac{\partial^2 \tau_{ij}}{\partial x_i \partial x_j} \,. \tag{3.17}$$

This form is particularly useful when dealing with mass and force sources in water and is used frequently in the present volume.

Interpretation

The wave equation, Eq. 2.52, can be derived directly from Eqs. 3.16 or 3.17 by assuming that there are no fluctuating mass inputs, that no fluctuating external forces are being experienced, and that the unsteady stress tensor is zero. In other words, these equations reduce to the wave equation in regions free of acoustic sources. The three terms on the right therefore represent the dominant types of sources of acoustic radiation. The first term on the right, involving unsteady mass flow into the fluid, acts basically as a monopole. The second term, the divergence of the unsteady forces applied at some boundary, is of dipole nature. It was these two types of sources which were considered by Stokes and Rayleigh. The last term, involving turbulent stresses in the fluid itself, is the term which Lighthill derived and showed to be of quadrupole nature.

In his classic article, Lighthill (1952) noted that there are three ways in which kinetic energy can be converted into acoustic energy:

1. by forcing the mass in a fixed region of space to fluctuate, represented by $\dot{q}'$;
2. by forcing the momentum in a fixed region to vary, i.e., by exerting a fluctuating external force on it, represented by *div* f'; or

3. by forcing the rates of momentum flux across fixed surfaces in space to vary, as by turbulent shear stresses in space.

The first two require boundaries, but the last can occur in open regions away from boundaries. Lighthill also recognized that the efficiencies of the terms as sources decreases with increasing dependence on spatial derivatives. One can understand this when it is recognized that for functions of the form $f(x - ct)$, which represent waves, a derivative with respect to time is of order of magnitude c greater than a spatial derivative. It follows then that, other factors being equal, the oscillating force term is small with respect to the mass flux term, and the shear term is the smallest. Lighthill's contribution was his pointing out that the lowest order source that could exist away from boundaries is of quadrupole nature, becoming efficient when fluctuating fluid velocities approach the speed of sound.

3.3 General Spherical Sources

Equation 3.17 indicates the nature of the common source terms found in acoustics, but its solution for a pressure field is often quite difficult. Solutions for many common sources are obtained by solving the source-free wave equation with appropriate symmetry and then matching the expression for particle velocity at the boundary to the vibratory velocity of the source, under the assumption that continuity of material requires that a fluid and its boundaries move in synchronism.

As indicated in Chapter 2, many sources exhibit spherical symmetry. In fact, it is possible to calculate the radiation field of any arbitrarily shaped source by superposition of the fields of small sources having spherical symmetry. By small sources, we mean sources small compared to a wavelength, i.e., sources having $ka << 1$, where k is the wave number and a is a characteristic dimension. Stated another way, the sound field from any arbitrary source can be calculated in terms of a superposition of elementary multipoles, provided only that the assumption of linearity is valid.

The basic properties of multipoles can be derived by considering the general problem of radiation from a small sphere whose surface vibrates in an infinite number of symmetrical modes. The general solution involves Legendre functions and spherical Bessel functions.* However, it can be shown that provided $ka << 1$ the radiation resistance and reactance are relatively simple functions of ka,

$$\sigma_{r_m} = \frac{(ka)^{2m+2}}{(2m+1)(m+1)^2\,(1 \cdot 3 \cdots (2m-1))^2} \tag{3.18}$$

$$\sigma_{x_m} = \frac{ka}{(2m+1)(m+1)}\,, \tag{3.19}$$

where m is the order of the multipole, starting with zero for a monopole. Since for small ka the reactance is large compared to the resistance, it follows that the radiation efficiency, Eq. 3.4, is given simply by

*See Morse (1948), Section 27, or Morse and Ingard (1968), Section 7.2.

$$\eta_{rad} \doteq \frac{\sigma_r}{\sigma_x} = \frac{(ka)^{2m+1}}{(m+1)\left(1 \cdot 3 \cdot 5 \cdot \cdots \cdot (2m-1)\right)^2}, \tag{3.20}$$

which is of the same form as Eq. 3.5 but includes values for the constants of proportionality.

3.4 Hydrodynamic Sources

Radiation Efficiencies

Many of the important sources that govern the noises of ships, submarines and torpedoes are hydrodynamic in nature, i.e., they are related in some way to the movement of a fluid past a vehicle or inside a conduit. These hydrodynamic sources of sound can each be classified in accordance with a major noise-producing mechanism: volume change (monopole), oscillating force (dipole), vibratory motions of small bodies (dipole), or free turbulence (quadrupole). The order of the multipole determines the Mach number dependence of the radiation process, and hence the order of magnitude of the radiation efficiency. Equation 3.20 gives the radiation efficiency in terms of the parameter ka. When a flow speed, U_o, exists, ka can be rewritten as a product of a dimensionless frequency and the Mach number:

$$ka = \frac{\omega a}{c_o} = \left(\frac{\omega a}{U_o}\right)\left(\frac{U_o}{c_o}\right) = \left(\frac{\omega a}{U_o}\right) M\ . \tag{3.21}$$

Using this form of ka, the radiation efficiencies of monopoles, dipoles and quadrupoles are given by the following:

$$\eta_{rad} = ka = \left(\frac{\omega a}{U_o}\right) M \quad (monopole)\ , \tag{3.22}$$

$$\eta_{rad} = \frac{1}{2}(ka)^3 = \frac{1}{2}\left(\frac{\omega a}{U_o}\right)^3 M^3 \quad (dipole) \tag{3.23}$$

and

$$\eta_{rad} = \frac{1}{27}(ka)^5 = \frac{1}{27}\left(\frac{\omega a}{U_o}\right)^5 M^5 \quad (quadrupole)\ . \tag{3.24}$$

These equations show increasing dependence on Mach number as the order of the multipole increases. The quantity in parentheses is a dimensionless frequency, which is usually of the order of unity.

Fluctuating-Volume Acoustic Sources

In situations for which the source of sound is associated with fluctuations of the total mass of fluid, Eq. 3.17 for acoustic pressure reduces to

$$\nabla^2 p' - \frac{1}{c_o^2}\ddot{p}' = -\dot{q}\ . \tag{3.25}$$

Since matter is not created within the fluid itself, any fluctuations of mass must occur at boundaries of the fluid region. Within the fluid itself there are no sources, the wave equation solution is valid, and the term on the right simply controls the amplitude of the acoustic signal. Provided the source is small compared to an acoustic wavelength, the solution of Eq. 3.25 is simply

$$p'(r,t) = \frac{\dot{Q}(t - r/c_o)}{4\pi r} = \frac{\dot{Q}(t')}{4\pi r} , \tag{3.26}$$

where t' is retarded time (Eq. 2.6) and

$$Q \equiv \int_V q \, dV = \frac{d}{dt}(\rho_o V) . \tag{3.27}$$

In liquids, density fluctuations are negligible, and Eq. 3.26 reduces to

$$p'(r,t) = \frac{\rho_o \ddot{V}(t')}{4\pi r} . \tag{3.28}$$

This is the most general form for sound radiation from a small fluctuating-volume (monopole) noise source. The strength of such a source is proportional to the product of fluid density and volume acceleration.

When they occur, fluctuating-volume noise sources radiate the highest levels found in hydroacoustics. This is because of the first-order dependence of the radiation efficiency on Mach number. Cavitation is an important source of monopole radiation in liquids. Pistons located in the boundaries also radiate as volume sources, as do pulsating pipe exhausts and certain tank resonances. Because of their prime importance, four chapters are devoted to volume noise sources.

Fluctuating-Force Sources

Any rigid surface acted on by a non-steady force will radiate sound. The reason is that there must be a fluctuating pressure field associated with any fluctuating force, and fluctuating pressure fields in a compressible medium radiate sound. The differential equation for sound generated by fluctuating forces is

$$\nabla^2 p' - \frac{1}{c_o^2}\ddot{p}' = \frac{\partial f_i'}{\partial x_i} = \nabla \cdot \vec{f}' . \tag{3.29}$$

In the absence of electromagnetic or chemical body forces, all forces are experienced at fluid boundaries and the solution of Eq. 3.29 for a concentrated force is

$$p'(\vec{r},t) = \frac{1}{4\pi r} \nabla \cdot \vec{F}(t') , \tag{3.30}$$

where $F(t')$ is the total fluctuating force. Relative to retarded time, the divergence of a function and its time derivative are of the same form and differ only by the speed of sound. In terms of the time derivative,

$$p'(\vec{r},t) = \frac{\dot{\vec{F}} \cdot \hat{r}}{4\pi r c_o} = \frac{\dot{F}(t')}{4\pi r c_o} \cos\theta \ , \tag{3.31}$$

where θ is the angle between the force vector and the direction to the field point for which the pressure is being calculated. The cos θ term represents a dipole pressure pattern.

Since it is virtually impossible to produce a steady force without also producing a fluctuating component, sound having dipole characteristics is invariably generated as a by-product of a useful force. From Eq. 3.23, the radiation efficiency of such a dipole source is proportional to the third power of the Mach number. Of the many fluctuating-force noise sources, those associated with propellers are usually dominant because the highest flow speeds occur at propeller blade-tip sections. The local flow speed at a propeller tip is the vector sum of the forward and rotational speed components. This tip speed is generally about three times the forward speed, resulting through third-power dependence on Mach number in dominance of the propeller tip sections relative to other parts of the vessel that experience only the forward speed. Fluctuating-force noises are quite important in underwater acoustics and are discussed in more detail in Chapter 9.

Turbulence Noise

Fluctuating-volume and -force noise sources generally occur at fluid boundaries. However, the source term in Lighthill's equation includes a component associated with hydrodynamic motions of the fluid itself. The non-steady stress tensor may be written

$$\tau'_{ij} \doteq (\rho v_i v_j)' + (p'_{ij} - p'\delta_{ij}) + (p' - c_o^2\rho')\delta_{ij} \ . \tag{3.32}$$

The first term represents fluctuating shear stresses associated with turbulent fluid motions, the second accounts for viscous stresses, and the third represents heat conduction and/or nonlinearity. At the Reynolds and Mach numbers of liquid flows, only the first term representing turbulence is important. To a first order,

$$\tau'_{ij} \doteq 2\rho_o U u'_i \ , \tag{3.33}$$

where u's have been used rather than v's to indicate that all fluctuating as well as steady velocities in the stress tensor refer to hydrodynamic quantities rather than acoustic ones.

By analogy with Eqs. 3.26 and 3.30, and assuming Eq. 3.33, the sound generated by turbulence is given by

$$p'(\vec{r},t) \doteq \frac{\rho_o U}{2\pi r} \frac{\partial^2}{\partial x_i \partial x_j} \int_V u'_i(t')\, dV \ , \tag{3.34}$$

where differences in retarded time must be considered unless the source region is small compared to a wavelength. The two spatial derivatives imply a basic quadrupole nature of this sound.

The radiation efficiency of fluid turbulence can be estimated from Eq. 3.24, using hydrodynamic relations for cold jets. The source radius, a, can be assumed to be equal to one half the scale length, ℓ, of the largest turbulent eddies; hence

$$\eta_{rad} \doteq \frac{1}{27}(ka)^5 \doteq \frac{1}{27}\left(\frac{1}{2}\right)^5\left(\frac{\omega\ell}{U_o}\right)^5 M^5 \; . \tag{3.35}$$

where U_o is the flow speed used in calculating the Mach number. Experiments with cold jets reveal that $\omega\ell \doteq U_o$, so that

$$\eta_{rad} \doteq \left(\frac{1}{27}\right)\left(\frac{1}{2}\right)^5 M^5 \doteq 1.1 \times 10^{-3} M^5 \; . \tag{3.36}$$

Since about one sixth of the fluid mechanical power in a wake or jet occurs as vibratory power of the turbulence, it follows that

$$\eta_{ac} \doteq 2 \times 10^{-4} M^5 \; , \tag{3.37}$$

which result is in good agreement with measurements of the noise powers of cold subsonic jets.

Noise from Wake Turbulence

When a body is propelled through water, a significant fraction of the total power is converted into wake turbulence, which eventually decays into heat. One might expect this turbulence to be a major source of sound, as it is in the case of a jet aircraft. However, the noises emitted by wake turbulence in water are entirely negligible provided there are no bubbles present.

It is clear from Eq. 3.37 why turbulence noise is of such importance in air and of so little importance in water. The difference is the Mach number. Jets in air often have Mach numbers in the vicinity of unity or even higher. In water, a 60-knot vehicle would have a Mach number of only 0.02. Only about one part in 10^{12} of its power would be radiated from the free turbulence of its wake. It is because of the low Mach number that quadrupole sources can be considered completely negligible in hydroacoustics, monopoles and dipoles at the boundaries always being dominant.

Sometimes noise does radiate from the wake of a vessel, but this noise is attributable to entrained air bubbles. Crighton and Ffowcs Williams (1969) have shown that monopole radiation resulting from the volumetric response of wake bubbles to turbulent pressure fluctuations overwhelms the quadrupole radiation by as much as 50 dB for a 1% concentration of air.

Flow Noise

Another way that fluid turbulence can be an important source of noise in liquids involves interaction with a boundary. Thus, the fluctuating pressures associated with a turbulent boundary layer excite flexural vibrations of the solid, and these vibrations then radiate sound. This flow noise occurs whenever fluids flow over non-rigid bodies or inside pipes and tubes. It is an especially important source of sonar self-noise.

Flexural wave radiation is a monopole process, and the efficiency of excitation of flexural waves by the fluctuating turbulent pressures is also proportional to the first power of the Mach number. Consequently, acoustic efficiencies for flow noise are proportional to the square of the Mach number. This topic has received a great deal of attention in the literature during the past 20 years, and a summary is given in Section 6.6.

Turbulence can also give rise to transducer self-noise when a receiver is placed in a turbulent stream. Pressure fluctuations associated with turbulence velocities cause *pseudo sound*, which though non-acoustic can be a dominant source of interference if the receiver is not protected from the flow by a dome.

3.5 Sources in Motion

In the previous section, radiation efficiencies of various hydrodynamic noise sources were related to Mach number, in the low Mach number limit. Although the noise was assumed to be a function of the mean flow speed, the calculation of the radiation itself assumed the source to be essentially at rest in the fluid medium. Steady motion of a source in a medium, and/or motion of a receiver relative to the medium, can affect received sound both as to its apparent frequency and its magnitude and oscillatory motions of constant masses and steady forces can create additional sound sources.

Doppler Shift

The most obvious effect of source motion relative to a receiver is a change of frequency known as the *Doppler shift*. The frequency received, f_a, is related to that radiated, f_s, by

$$f_a = \frac{f_s}{1 - M_S \cos\theta} , \tag{3.38}$$

where M_S is the convection Mach number of the source and θ is the angle between the motion vector and the direction toward the receiver. For small Mach numbers

$$f_a \doteq f_s (1 + M_S \cos\theta) , \tag{3.39}$$

which is the expression generally found in elementary texts.

If the receiver is in motion toward the source, then the apparent frequency will also be altered:

$$f_a = f_s (1 + M_R \cos\theta) . \tag{3.40}$$

Thus, for low Mach numbers motion of the receiver is equivalent to motion of the source.

Effect of Steady Motion on Level

Not only is the received frequency altered by steady motion of a source or receiver, but also the received signal is altered in strength from that calculated for a stationary situation. Lowson (1965) has shown that calculated values for both monopole and dipole type sources are modified by $(1 - M \cos\theta)^{-2}$, and that for quadrupoles the exponent changes from -2 to -3. Since this effect becomes significant only when relative Mach numbers exceed about 0.1, it is generally not included in underwater sound calculations.

Periodic Motions

The source terms in Eqs. 3.16 and 3.17 are expressed as partial derivatives and are calculated for volume elements fixed in the chosen coordinate system, in the Eulerian sense. There are therefore two distinct physical ways in which acoustic source terms can arise. One of these is fluctuations of mass, force and stress at positions fixed in a coordinate system. The other is oscillations of non-fluctuating quantities in position. Stated another way, both time and space changes of mass, force and stress produce acoustic disturbances. Of the two, fluctuations with time are generally more important than oscillations of position, especially in underwater acoustics. The reason is that motions in space must occur at speeds comparable to the speed of sound to be effective as sound radiators. It can be shown that a constant mass experiencing oscillatory motion radiates with the same directional pattern and Mach number dependence of radiation efficiency as a dipole, while a constant force executing periodic motion radiates as a quadrupole. Each is one order multipole higher than for the corresponding fluctuating quantity.

For many years, explanations of tonal radiation from rotating propellers were based on the periodic motion of steady forces, as originally derived by Gutin (1936). Gutin's results exhibit a very strong Mach number dependence and predict strong tones only at high Mach numbers. However, as discussed in Chapter 1, it is impossible to produce steady mechanical forces free of vibratory values. It is now recognized that even in air fluctuating forces are usually dominant sources of tonal radiation. In water, periodic motions of steady forces are never important, and Gutin's analysis is therefore omitted from this volume.

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