The Time Variable Acoustic Propagation Model (TV-APM)

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1 Introduction

The Time Variable Acoustic Propagation Model (hereafter TV-APM) aims at generating time variable simulated acoustic channel responses between moving sources and receivers in a realistically modelled environment. The main objective of this simulator is to properly take into account the Doppler effects induced by source-receiver relative motion as well as the effects of that motion that propagate through the acoustic channel. The TV-APM was particularly conceived to address acoustic communications within moving nodes of an underwater network so its computational efficiency at high frequencies was optimized. The TV-APM simulator was developed as part of Deliverable 2.1 of project UAN - Underwater Acoustic Network funded under the 7th European Framework Program, contract No. 225669 (webpage: http://www.siplab.fct.ualg.pt/proj/uan.shtml).

The current version of the TV-APM is provided as a compact zip file, containing a set of Matlab M-files and configuration data (*.dat) files; the M-files define configuration parameters used by the TV-APM, load the configuration data and call the Bellhop ray tracing model [?]-[?] to simulate dynamic signal propagation in an underwater environment under the following conditions:

- Source and array can be placed anywhere inside a three-dimensional box; such box is bottom limited by a user-specified bathymetry and (if desired) by a wind-driven surface wave.
- Linear velocities can be attributed to both source and array and the corresponding positions are updated progressively along transmissions. When a surface wave is specified its state is updated along transmissions as well.
- A sound speed profile can be specified by the user.
- A transmitted signal can be specified by the user.

At the output the TV-APM provides the channel Impulse Responses (IRs) along time as well as the output signals obtained with a time-variable convolution. A general discussion of the web-based TV-APM can be found in [?]-[?]. As provided the current "standalone" (as opposite to the web-based) TV-APM is ready for a preliminary "blind" run using a default set of configuration data and parameters; users are expected to write their own configuration data and parameters in order to run the TV-APM for their particular needs. The TV-APM is distributed in the hope that it will be useful, but without any warranty. Users are invited to correct bugs, extend features and share their contributions with the community. This manual is organized as follows: section ?? describes the configuration data and parameters required to run the TV-APM; running of the TV-APM and reading of the output files is discussed in sections ?? mad ?? respectively; the preliminary "blind" run of the TV-APM is described in section ??; the conclusions are presented in section ??.

2 Configuration data

The TV-APM depends on configuration *.dat files to define the sea surface, the bottom bathymetry, the sound speed profile and the transmitted signal; configuration parameters are defined in the M-file parameters.m. The TV-APM expects all data and parameters to be given in the following units: distances and depths are to be given in meters, velocities and sound speeds in m/s, densities in g/cm³, angles (relative to the X axis) in degrees and attenuations in dB/ λ . The user is allowed to change the *.dat and parameters.m files accordingly with the sub-sections of this section and taking in consideration the Bellhop manual [?]. The *.dat and parameters.m files are described in the following sections.

2.1 The sea surface

The initial state of the sea surface is defined in the file surface.dat; the structure of this file is organized as follows:

2 % Two points along X x1 x2 % First and second coordinates along X 2 % Two points along Y y1 y2 % First and second coordinates along Y 0.0 0.0 % 2x2 matrix of zero surface 0.0 0.0 % elevation The pairs (x1,y1), (x2,y1), (x1,y2), (x2,y2) define the four corners, in Cartesian coordinates, of the upper box boundary. The information contained in this file will be overwritten automatically if the user chooses to change the bathymetry or to generate a wind-driven surface wave (see sections ?? and ?? respectively). In the current version of the TV-APM either a flat or a wind-driven surface wave is considered, i.e., static non-flat surfaces are not supported by the TV-APM.

2.2 The bathymetry

The bathymetry is defined in the file **bathymetry.dat**; the general structure of the bathymetry file is organized as follows:

```
M % Number of points along X
x1 x2 ... xM % Coordinates along X
N % Number of points along Y
y1 y2 ... yN % Coordinates along Y
z11 z12 ... z1M % NxM matrix
z21 z22 ... z2M % of bottom depths
.... %
zN1 zN2 ... zNM %
```

where the values of x and y define a rectangular grid, on which the values of depth z are defined. Generally speaking bathymetry takes precedence over surface; therefore, the corners defined in **bathymetry.dat** overwrite the corners defined in **surface.dat**; if the user chooses to generate a wind driven surface wave the corresponding corners of the wave will be fitted to the corners of bathymetry.

2.3 The sound speed profile

The sound speed is defined in the file **sound_speed.dat**; the general structure of the sound speed file is organized as follows:

```
z1 c1 % first depth, first value of sound speed
z2 c2 % second depth, second value of sound speed
....
zM cM % Last depth, last value of sound speed
```

Particular care should be taken by the user in order to define a sound speed, which spans till the deepest value indicated in the bathymetry file. Otherwise, potential failure of the TV-APM can occur wherever Bellhop is requested to calculate travel time data along a transect, deeper than defined in sound_speed.dat. Additionally, if the initial depth is above 0 m another pair of points with the values (0,c1) is added to the sound speed data. Special care should be taken for a proper discretization of the sound speed profile: a too sparse discretization can fail to model accurately the profile, while a too dense discretization will induce the generation of unrealistic impulse responses between model runs. Currently accounting for a sound speed field is not supported by the TV-APM.

2.4 The transmitted signal

The transmitted signal defined in the file signal.dat is in fact the *analytic* signal $x_a(t)$ in the baseband of the real-valued signal emitted by the acoustic source x(t):

$$x_a(t) = x(t) + i\hat{x}(t) , \qquad (1)$$

where $\hat{x}(t)$ stands for the Hilbert transform of x(t). Thus, the user is expected to produce his own complex baseband representation $x_a(t)$, of the time-based transmitted signal x(t). Since $x_a(t)$ is a complex-valued function (defined in the time domain as x(t)) the transmitted signal defined in signal.dat has the following structure:

```
rxa1 ixa1 % first real part of xa, first imaginary part of xa
rxa2 ixa2 % second real part of xa, second imaginary part of xa
....
rxaM ixaM % Last real part of xa, last imaginary part of xa
```

The transmitted signal is defined in the M-file UnderProSim.m after loading the data from signal.dat as

```
x = signal(:,1) + sqrt(-1)*signal(:,2);
```

Thus, the total duration of transmissions is defined in the M-file UnderProSim.m as

tmax = 1 + length(x)/fs_x; % total duration of the scenario

where fs_x is defined in the M-file parameters.m.

2.5 Configuration parameters

Additional information regarding the initial positions of both source and array, respective linear velocities, and also parameters that define the properties of the propagating wind induced sea surface wave (if requested), is stored in the M-file parameters.m, which has the following structure:

```
%______
% source
source_x = [... ...]; % source initial position (x,y,z) (m)
source_v = [... ...]; % source velocity (vx,vy,vz) (m/s)
source_nrays = ...; % number of propagation rays considered
source_aperture = ...; % maximum launching angle (degrees)
source_ray_step = ...; % ray calculation step (meters), modified later
% transmitted signal
%______
fc = ....; % input signal carrier
                             frequency (Hz)
fs_x = ....; % input baseband signal sampling frequency (Hz)
% Bottom
bottom_properties = [... ...];
%bottom_properties(1) = compressional speed (m/s)
%bottom_properties(2) = bottom density (g/cm3)
%bottom_properties(3) = bottom attenuation (dB/wavelength)
% Array
%______
array_x = [... ... ...]; % receivers array initial position (x,y,z) (m)
array_v = [... ...]; % receiver array velocity (vx,vy,vz) (m/s)
first_hyd = ...; % depth of the first hydrophone (m)
last hyd = ...; % depth of the last hydrophone (m)
               % depth of the last hydrophone
last_hyd = ...;
                                     (m)
delta_hyd = ...;
               % separation between hydrophones (m)
%______
% Wind induced sea surface wave
%______
U
      =
         ...; % wind speed
    =
         ...; % direction of propagation in degrees
theta
spreading = '...'; % options: 'none', 'mits', 'hass'
```

where the meaning of the different parameters should be straightforward. For a better understanding of the parameters that define the properties of the sea surface wave a detailed theoretical description of such parameters is given in appendix ??. Before proceeding with calculations the TV-APM will test for any situation in which either the source or the array might be positioned out-of-the-box for any given choice of source and array positions and velocities. When such a situation is detected a warning is issued to the user and calculations are stopped. The user should be aware that too large velocities (mainly in the z direction) will cause a large Doppler effect and a large amount of impulse responses to be generated by Bellhop. Such situation can enlarge strongly the run time and the memory space needed to store the TV-APM outputs. The following features are not supported by the current version of the TV-APM:

- Shear bottom properties and range dependence of bottom properties.
- Non-linear velocities of the source or the array.
- Non-linear arrays.
- Swells (although a swell.m M-file is provided).

3 Running the TV-APM

Once the configuration data and parameters had been specified the user can run the TV-APM within Matlab by entering the command

>> UnderProSim

After reading and checking the configuration data and loading parameters the command produces several plots, showing the positions of the source and the receiver and creates a structure, named Scenario_info, which is passed as argument to the function OceanTVIR; this function is then executed and performs a set of calculations (scaled in several iterations), generating travel time and amplitudes calling Bellhop for the different positions of the source and the array; each Bellhop call relies on the calculation of the corresponding transects along both surface and bottom. After each iteration OceanTVIR checks for the presence of aliasing in the Doppler spectrum by analyzing the spreads in the spreading function; if the incoherence is above a given threshold a new set of calculations is performed (such threshold is defined in the M-file OceanTVIR.m with the criterium_threshold parameter); to this end old and new calculations are merged by calculating new states at time intervals between each pair of old states. Calculations are finished if the aliasing of Doppler spreads is below the threshold or if the maximal number of iterations is exceeded.

4 General outputs

At the end of execution a text file named computation_advance.txt is produced, containing general information about the calculations. Additionally, two mat files are written, named Results.mat and Results_surface.mat; the first one contains the channel impulse response, calculated using the transmitted signal and travel time and amplitude data calculated by Bellhop; the second one contains the initial state of the wind driven sea surface. The Results.mat can be read in the Matlab prompt executing the command

>> display_results

which asks for the hydrophone number. After inputing a proper number the functions shows the corresponding channel impulse response, together with its associated functions. A hydrophone number of 0 or a number higher than the number of available hydrophones stops calculations. A detailed discussion of the sea surface impact on Doppler spreads can be found in [?], while applications for underwater communications are discussed in [?].

5 Preliminary run

As provided the current version of the TV-APM is ready for a preliminary run using a default set of configuration data and parameters, which are described in detail in this section; the data was taken in the proximity to the Elba island. The default data and parameters allow to reproduce the result presented in section **??**.

5.1 Default data

5.1.1 Sea surface

The default data contained in **surface.dat** corresponds to the following:

```
2
-10000 10000
2
-5000 20000
0.0 0.0
0.0 0.0
```

5.1.2 Bathymetry

The default data contained in **bathymetry.dat** corresponds to the following:

```
143

1.0000e+00 ... 5.4822e+03

107

1.0000e+00 ... 1.7818e+03

...

4.8143333e+01 ...

... 3.2653649e+01
```

Such bathymetry is shown in Fig.??.



Figure 1: Default bathymetry provided in the bathymetry.dat file.

5.1.3 Sound speed profile

The default sound speed profile contained in **sound_speed.dat** corresponds to the following:

0.000000e+00	1.4898272e+03
1.000000e+00	1.4898269e+03
2.000000e+00	1.4898795e+03
3.000000e+00	1.4899786e+03
2.000000e+02	1.4890000e+03
2.0100000e+02	1.4890000e+03

Such profile is shown in Fig.??.



Figure 2: Default sound speed provided in the sound_speed.dat file.

5.1.4 Transmitted signal

The default transmitted signal contained in **signal.dat** corresponds to the following:

0.0000000e+00 0.0000000e+00 4.4719269e-12 -1.7081502e-08

-1.6098904e-10	6.8325746e-08
-1.7081493e-08	-1.7887704e-11
0.0000000e+00	0.000000e+00

It represents the analytic signal of a chirp between 8.75 kHz and 11.25 kHz.

5.2 Default parameters

The default set of parameters given in the M-file parameters.m corresponds to the following values:

```
% source
source_x = [1900 900 5]; % source initial position (x,y,z) (m)
source_v = [0 0 0]; % source velocity (vx,vy,vz) (m/s)
source_nrays = 2001; % number of propagation rays considered
source_aperture = 75; % maximum launching angle (degrees)
source_ray_step = 10;  % ray calculation step (meters), modified later
% transmitted signal
= 25600; % input signal carrier frequency (Hz)
fc
fs_x = 10000; % input baseband signal sampling frequency (Hz)
%______
% Bottom
bottom_properties = [1465 1.5 0.06];
%bottom_properties(1) = compressional speed (m/s)
%bottom_properties(2) = bottom density
                         (g/cm3)
%bottom_properties(3) = bottom attenuation (dB/wavelength)
% Array
array_x = [2800 400 0]; % receivers array initial position (x,y,z) (m)
array_v = [0 0 0]; % receiver array velocity (vx,vy,vz) (m/s)
first_hyd = 30;
              % depth of the first hydrophone (m)
            % depth of the last hydrophone
last_hyd = 60;
                                   (m)
delta_hyd = 2;
              % separation between hydrophones (m)
```

Thus, not only the source and the array are both static, as no wind driven wave is generated as well. For the given value of baseband sampling frequency transmissions last between 0 and 1.6 seconds.

5.3 Results

The results for the first hydrophone obtained after running the TV-APM with the default data and parameters are shown in Fig.??. Since for the default set of parameters no source or array are moving and the surface is flat the impulse response exhibits a set of straight lines, to which correspond zero Doppler spreads.

6 Conclusions and future work

The TV-APM is a valuable tool to discuss arrival scattering due to the propagation of wind-driven sea surface waves or relative motion between a source and an array. Modelling in the future can be oriented along the following guidelines:

- Accounting for range dependent bottom properties, shear included.
- Accounting for sound speed fields and sound speed temporal variability, in particular using empirical orthogonal functions.
- Considering the statistical distribution of travel times and amplitudes.
- Considering the inclusion of additional acoustic models.

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Figure 3: Default case: output for the first hydrophone from Results.mat.

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A Wind driven surface waves

Wind driven surface waves can be characterized by the Pierson-Moskowitz spectrum [?, ?]:

$$S(f) = \alpha \frac{g^2}{\left(2\pi\right)^4 f^5} \exp\left[-\frac{5}{4} \left(\frac{f_m}{f}\right)^4\right]$$
(2)

where

$$f_m = 0.13 \ \frac{g}{v} \tag{3}$$

represents the peak's frequency, v is the wind velocity, $g = 9.8 \text{ m/s}^2$ and $\alpha = 8, 1 \times 10^{-3}$ is Phillip's constant; the quantity

$$T_p = \frac{1}{f_m} = \frac{v}{0.13g} \tag{4}$$

represents the peak's period. The Pierson-Moskowitz spectrum is intended to represent a fully developed wind generated sea and it is valid in the interval 10 m/s < v < 20 m/s. Another alternative is given by the JONSWAP spectral model [?]:

$$S(\omega) = \alpha' g^2 \omega^{-5} \exp\left[-\frac{5}{4} \left(\frac{\omega}{\omega_p}\right)^{-4}\right] \gamma^{\delta} , \qquad (5)$$

where δ is a peak enhancement factor:

$$\delta = \exp\left[-\frac{(\omega - \omega_p)^2}{2\sigma_0^2 \omega_p^2}\right] .$$
 (6)

The parameters γ and σ_0 are given as $\gamma = 3.3$, $\sigma_0 = 0.07$ for $\omega \leq \omega_p$ and $\sigma_0 = 0.09$ for $\omega > \omega_p$, while α' is function of fetch X and wind speed v:

$$\alpha' = 0.076 \left(\frac{gX}{v}\right)^{-0.22} , \qquad (7)$$

and peak frequency ω_p is given by

$$\omega_p = 7\pi \left(\frac{g}{v}\right) \left(\frac{gX}{v^2}\right)^{-0.33} . \tag{8}$$

A.1 Wave height and speed

Surface wave height can be related to wind speed through the well-known Beaufort scale. For deep water propagation the phase speed of the surface wave is calculated as

$$c_p = \frac{g}{\omega_p} \tag{9}$$

while the group speed corresponds to

$$c_g = \frac{1}{2}c_p \ . \tag{10}$$

A.2 Frequency to wavenumber conversion

The spectra described are defined in the frequency domain. For acoustic propagation is preferable to generate realizations of the sea surface in space rather than in time, which requires the spectra to be defined in the wavenumber domain. The conversion from frequency to wavenumber can be accomplished by taking into account that

$$E(k)dk = S(f)df , \qquad (11)$$

where S(f) represents the spectrum in the frequency domain and E(k) represents the spectrum in the wavenumber domain. For surface waves over deep oceans frequency and wavenumber can be related as

$$\omega = \sqrt{gk} , \qquad (12)$$

which allows to obtain the relationship

$$\frac{df}{dk} = \frac{1}{4\pi} \sqrt{\frac{g}{k}} \ . \tag{13}$$

Thus, the spectrum in the wavenumber domain can be calculated from the spectrum in the frequency domain as

$$E(k) = \frac{1}{4\pi} \sqrt{\frac{g}{k}} S(f) . \qquad (14)$$

Generally speaking the peak spectrum shifts towards lower wavenumbers (i.e. longer wavelengths) as wind speed increases. Therefore, high values of wind speed induce the propagation of low-frequency surface waves, associated with larger amplitudes and more energy transfer to the ocean.

A.3 Angular spreading

The directivity of wind speed can be incorporated into the modeling by introducing a spreading function along different angles; the full spectrum can then be written as

$$F(k,\theta) = E(k)D(f,\theta) , \qquad (15)$$

where $D(f, \theta)$ represents the spreading function. Since the total energy in the directional spectrum should be the same as the total energy in the onedimensional spectrum a fundamental condition for the choice of the function $D(f, \theta)$ is that

$$\int_{-\infty-\pi}^{\infty} \int_{-\pi}^{\pi} E(k)D(k,\theta) \ d\theta \ dk = \int_{-\infty}^{\infty} E(k) \ dk \tag{16}$$

Some of the most common spreading functions are presented in the following sections.

A.3.1 Cosine-squared spreading function:

The cosine-squared sea surface spreading (which is independent of frequency) is given by the expression [?, ?]

$$D(\theta) = \begin{cases} \frac{2}{\pi} \cos^2(\theta - \theta_0) & -\frac{\pi}{2} + \theta_0 < \theta < \frac{\pi}{2} + \theta_0 ,\\ 0 & \text{otherwise} , \end{cases}$$
(17)

where θ_0 represents the wind's direction. The cosine-squared spreading function is strongly non-isotropic and different from zero only in the region where $|\theta - \theta_0| < \pi/2$.

A.3.2 Mitsuyasu spreading function:

The Mitsuyasu sea surface spreading can be written compactly as [?]

$$D(f,\theta) = \frac{\Gamma(s+1)}{2\sqrt{\pi}\Gamma(s+1/2)} \left[\cos^2\left(\frac{\theta-\theta_0}{2}\right)\right]^s , \qquad (18)$$

where $\Gamma(s)$ represents the Gamma function; the parameter s controls the angular distribution of energy along frequency as follows:

$$s = \begin{cases} 9,77 \left(f/f_m \right)^{-2,5} & f \ge f_m \\ 6,97 \left(f/f_m \right)^5 & f < f_m ; \end{cases}$$
(19)

the definition of s reflects the increase of the parameter near the peak spectral frequency and its decrease at low frequencies.

A.3.3 Hasselmann spreading function:

The Hasselmann sea surface spreading can be written compactly as [?, ?]

$$D(f,\theta) = \frac{1}{N_p} \left[\cos^2 \left(\frac{\theta - \theta_0}{2} \right) \right]^p ; \qquad (20)$$

through a maximum likelikood technique collected data was fitted to analytical expressions in order to obtain the following dependence of the parameters on frequency:

$$p = 9,77 \left(\frac{f}{f_m}\right)^{\mu} \tag{21}$$

and

$$\mu = \begin{cases} 4,06 & f < f_m ,\\ -2,34 & f > f_m ; \end{cases}$$
(22)

the normalization factor corresponds to

$$N_p = 2^{1-2p} \pi \frac{\Gamma(2p+1)}{\Gamma^2(p+1)}$$
(23)

where $\Gamma(p)$ stands again for the Gamma function.

B Swells

A swell is a long-wavelength surface wave, which is far more stable in his direction and frequency than normal wind-induced surface waves. Swells are often created by storms thousands of nautical miles away from the areas where they are observed. Such large distances allows the waves comprising the swells to become more stable and clean as they travel toward the coast. The level of energy contained in swells is influenced by the following factors: wind velocity, wind area (i.e. the amount of ocean surface area which is affected by wind blowing in the same direction, also known as fetch), and wind duration. During a typical open ocean winter storm wind speed can achieve up to 23.15 m/s, blowing over 1000 km for 36 hours will produce a periodic swell with a characteristic period of 17-20 s. Swell characteristics can be predicted using ocean surface models as the NOAA (National Oceanic and Atmospheric Administration) ocean surface model WAVEWATCH III. Currently the TV-APM does not account for swell modelling (although a swell M-file is provided).

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