

## A Reverberation Calculation Using Stepwise Coupled Modes

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### Abstract

A sample reverberation problem is devised that has some of the characteristics that might be seen in monostatic reverberation from a uniform scattering layer. The problem consists of a point source on the axis of a cylindrically symmetric annulus of square wave bottom corrugations. The reverberation is obtained using Fourier synthesis of a Gaussian pulse and the stepwise coupled mode field. Modifications to the stepwise coupled mode method that facilitate the calculations are discussed. Results of the reverberation calculations are analyzed in regards to time of arrival and pulse dispersion.

### Introduction:

The term reverberation means “to continue in a series of echoes.” The echoes, in an underwater acoustic waveguide, return from all directions and subsequent times; but it is not practical find a comprehensive solution of the three dimensional wave equation in the time domain. A lower dimensional problem may be of interest if it has physical significance. Two dimensional problems, with linear or cylindrical symmetry, are potential candidates since their solution is possible using stepwise coupled modes [1 and 2] and Fourier synthesis.

Existing reverberation models [3, 4 and 5] sometimes employ a uniform scattering layer as a source of reverberation. They deal with both monostatic and bistatic geometries, with collocated and separated sources and receivers. The bistatic geometry retains out of plane scattering that violates linear and cylindrical symmetry. The monostatic geometry, with a uniform scattering layer, is close, but not identical, to cylindrical symmetry. The distinctions are as follows. The existing models of reverberation in the monostatic geometry discard out of plane scattering while none exists in the two dimensional cylindrically symmetric stepwise coupled mode result. Existing models of monostatic reverberation assume that different azimuthal directions add incoherently where as the two dimensional cylindrically symmetric stepwise coupled mode result is fully coherent. Nothing is random. Finally, existing models of monostatic reverberation ignore the multiple scattering that is present in the stepwise coupled mode result.

The foregoing considerations led to the proposal of Test Case 5 in the Reverberation and Scattering Workshop held at Gulfport, Mississippi on May 2-5, 1994. The test case consisted, in part, of a point source on the axis of a cylindrically symmetric annulus of depth corrugations. A train of echoes, from the annulus, should be received at the source range. The complications associated with the corrugations being repeated on the entire range interval are avoided.

### **Stepwise Coupled Modes:**

The single frequency stepwise coupled mode method represents a two dimensional range variation as a sequence of locally range independent regions in which the normal mode solution is valid. A perfectly reflecting boundary condition at great depth and zero depth provides an discretization of the modal spectrum. The cylindrically symmetric solution, in each region, is represented as a truncated sum of terms containing cylindrical waves propagating in opposite directions whose range dependence is described by Hankel functions of order zero type one and two. The depth dependence is given by the local normal modes. The sum contains two sets of unknown coefficients for each region. These unknown coefficients, in adjacent regions, satisfy a linear relation brought about by the continuity of pressure and radial particle velocity at vertical interfaces. In addition, the unknown coefficients are chosen to satisfy the radiation condition at large range and the source condition at zero range.

Two modifications of the existing [6 and 7] stepwise coupled mode code were made to accommodate the reverberation calculations to be described. The homogeneous bottom layer was replaced by a refracting bottom using the Galerkin method [2]. The previously existing code used a homogeneous absorbing layer bounded by a perfect reflector to represent an absorbing half-space. This approach requires a lot of modes because the steep angle modes try to coalesce as the thickness of the absorbing layer is increased. The current code is more efficient because fewer modes are needed when the absorbing half-space is represented by a region in which the attenuation is allowed to ramp up to some large value at the perfect reflector. The ramping absorber is important when it comes to checking for convergence because it partially decouples the thickness of the absorber from the number of modes.

The other modification is the explicit calculation of the scattered field in the range independent region containing the source (source region). The total field

$$P_{tot}(r, z) = \hat{\mathbf{A}} \sum_{m=1}^M [a_m H_0^{(1)}(k_m r) + b_m H_0^{(2)}(k_m r)] \mathbf{f}_m(z) \quad (1)$$

is not finite at range zero because both the Hankel function have logarithmic singularities at range zero. The source condition requires that the total field have a specified singularity at zero range, associated with the delta function source, and that the remaining component of the total field be regular (bounded) at zero range. This is accomplished by the source condition

$$a_m = S f_m(s) + b_m \quad (2)$$

involving the unknown coefficients in source region for a source of strength of  $S$  at depth  $s$ .

The source condition in Eq. (2) is substituted into Eq. (1) to yield

$$P_{tot}(r, z) = s \sum_{m=1}^M \hat{\mathbf{A}} f_m(z) f_m(s) H_0^{(1)}(k_m r) + \sum_{m=1}^M \hat{\mathbf{A}} 2b_m f_m(z) J_0(k_m r) \quad (3)$$

As a result, the total field is divided into a singular part, which is the usual outgoing normal mode solution involving the Hankel function of order zero type one; and the scattered part which is a standing wave in the source region involving the Bessel function of order zero. The scattered field in the source region is

$$P_{scatt}(r, z) = \sum_{m=1}^M \hat{\mathbf{A}} 2b_m f_m(z) J_0(k_m r) \quad (4)$$

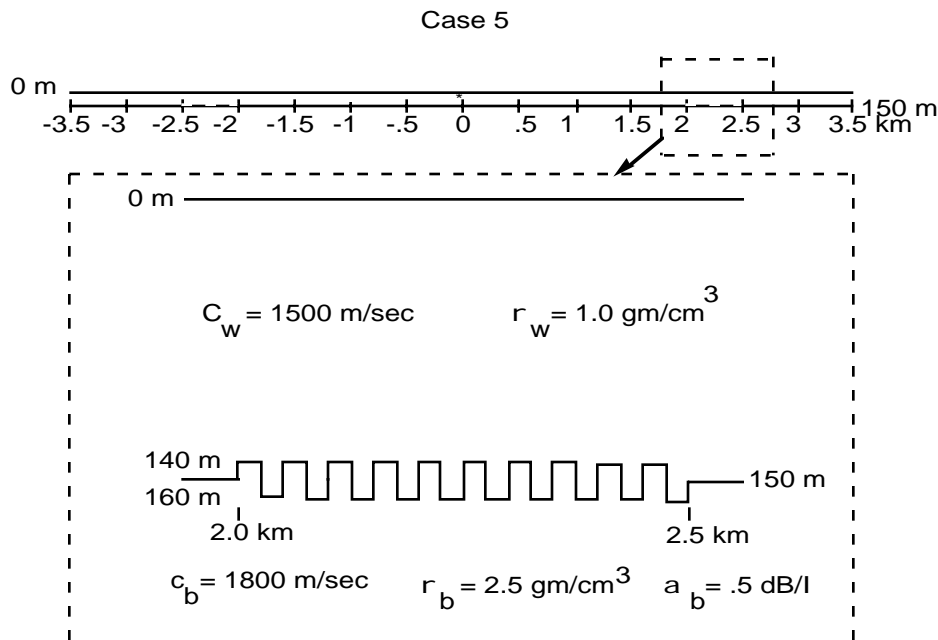
The scattered field has always been implicitly available. Now the code actually computes the scattered field and writes it on a separate output file. The scattered field is not computed outside the source region.

The time history synthesized from the input pulse spectrum and the scattered part of the total field at range zero in Eq. (4) is the monostatic reverberation.

### **Test Case Analysis:**

The analysis to follow is devoted to part of Test Case 5 of the Reverberation and Scattering Workshop. The scattering and reverberation, in this problem, arises from a circular annulus of corrugated bottom as show in Fig. 1. The corrugated bottom is

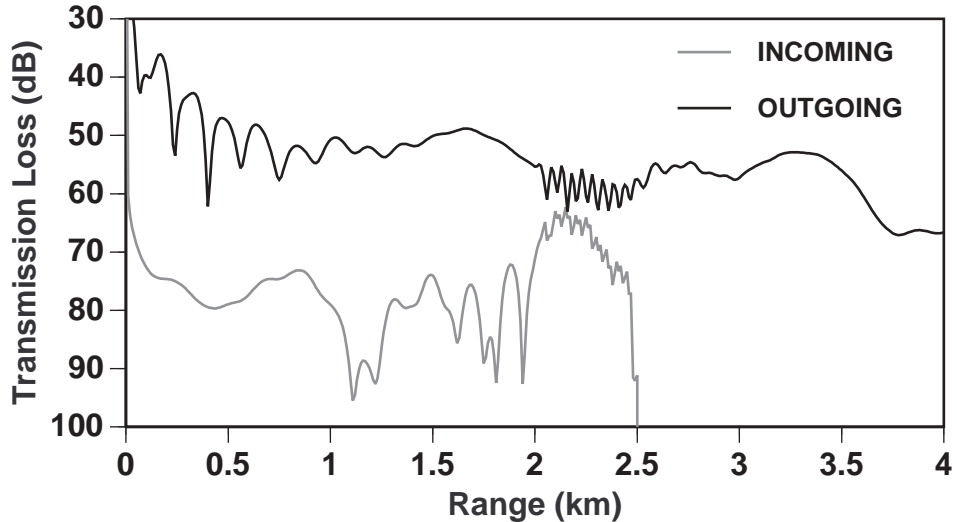
described by a square wave with an amplitude of 10 meters about a mean water depth of 150 m with 10 cycles in the range interval 2.0 to 2.5 km. A total of 22 regions are used to define the problem. The water layer has a sound speed of 1500 m/sec and a density of  $1.0 \text{ g/cm}^3$ . The bottom layer has a sound speed of 1800 m/sec, a density of  $2.5 \text{ g/cm}^3$  and an attenuation of .5 dB/wave length. The 50 m period of the square wave is commensurate with the wave length at 30 Hz. The source was at a depth of 50 m and the receiver was at the same range as the source but at a depth of 45 m. This is essentially a monostatic configuration



**Figure 1.** Schematic of the environment of Test Case 5. The area where the corrugated bottom is located is shown in an expanded scale.

To check convergence, a single frequency calculation at 30 Hz was performed using 120 modes. The absorbing half-space, starting at a depth of  $150 \pm 10$  m, was modeled as homogeneous to a depth of 1500 m and then the attenuation was linearly increased from 0.5 dB/wave length to 10 dB/wave length at 2000 m where a pressure release boundary condition was applied. Convergence was assessed using the backscattered energy. The same calculation, with 130 modes, produced a negligible change in the backscattered energy. A plot of the outgoing and ingoing parts of the 30 Hz field at the 45 m receiver on the range interval 0 to 4 km is shown in Fig. 2. A similar plot of the total and scattered field at 30 Hz is shown in Fig. 3. The same check was

performed at 45 Hz where 140 modes were required. A check of the artificial absorber was performed at 15 Hz and it was found to be adequate when compared to a calculation with a homogeneous artificial absorber extending to 3000 m.



**Figure 2.** Transmission Loss (dB) versus Range (km) for Test Case 5. The solid line is the outgoing result and the gray line is the ingoing field for a frequency of 30 Hz and a source depth of 50 m and a receiver depth of 45 m.

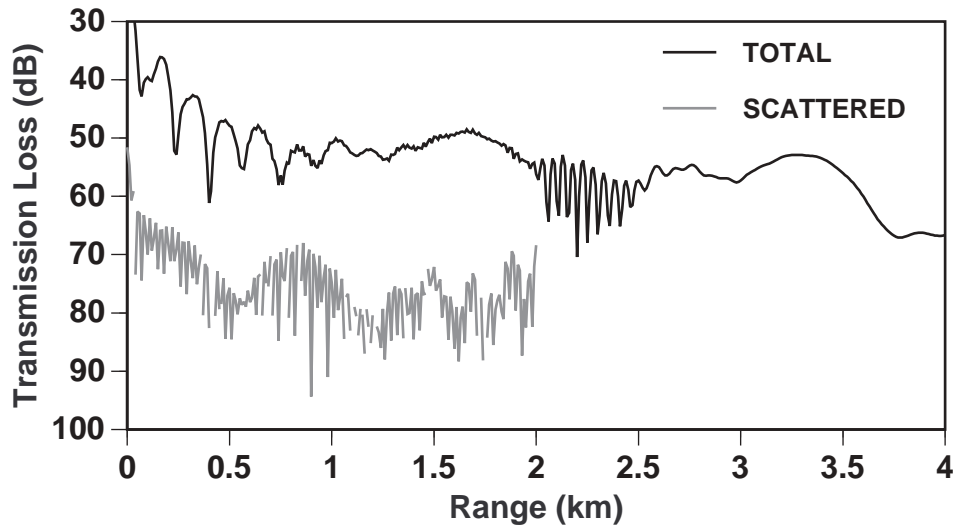
The pulse calculation employed a Gaussian source spectrum. The source spectrum was

$$F(w) = e^{-[(w-m)\xi s]^2} + e^{-[(w+m)\xi s]^2} \quad (5)$$

where  $w$  is in rad/sec,  $2pm = 30$  Hz and  $2ps = 10$  Hz. The factor of 1/2 that normally appears in the Gaussian distribution is omitted. The exact Fourier transform of Eq. (5) is

$$f(t) = (2\sqrt{ps})\cos(2pmt)e^{-(ps t)^2} \quad (6)$$

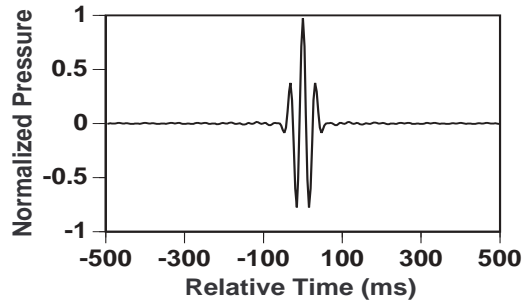
which is a Gaussian centered at time  $t = 0$  modulated by a 30 Hz carrier.



**Figure 3.** Transmission Loss (dB) versus Range (km) for Test Case 5. The solid line is the Total result and the gray line is the scattered field for a frequency of 30 Hz and a source depth of 50 m and a receiver depth of 45 m

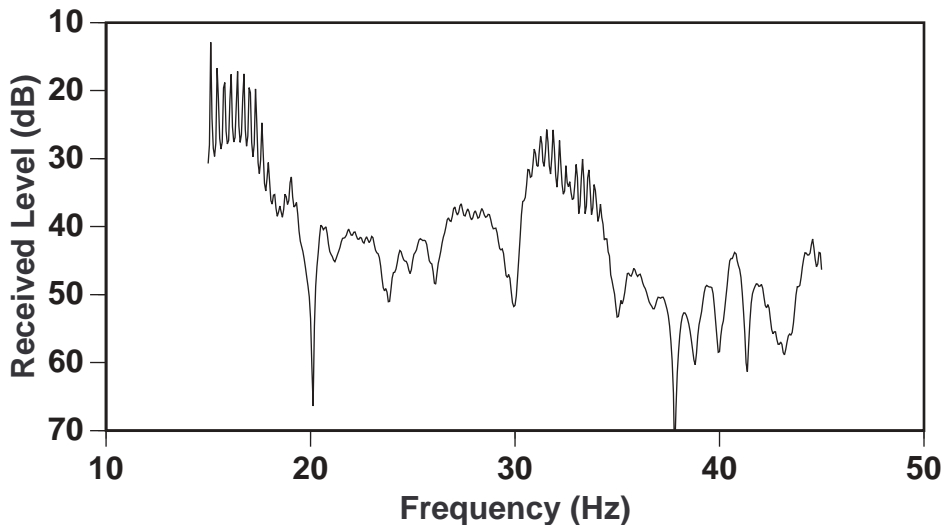
The pulse spectrum was divided by  $2\sqrt{ps}$  to produce a maximum amplitude of one in the time domain. The input pulse has a width of about 0.15 sec. Its reconstruction, using 1 Hz sampling, by a discrete Fourier transform is shown in Fig. 4. The spectrum is set to zero outside the frequency intervals  $[-15, -45]$  and  $[+15, +45]$  in the reconstruction.

The discrete Fourier synthesis of the reverberation is done using 1/16 Hz sampling over the same frequency intervals described above. The calculation of the stepwise coupled mode fields at 481 frequencies took approximately 28.3 hours of cpu time on a Cray YMP. The synthesized time series is periodic, with a period of 16 sec. When the reverberation is synthesized from the input pulse spectrum and the spectrum of the scattered field at the monostatic receiver, the result is equivalent to the reverberation from a train of input pulses, repeated every 16 sec. If the entire reverberation series is over by the end of 16 sec, then the pulses input at -16 sec and earlier do not appear in the 0 to 16 sec time interval.



**Figure 4.** Time History of gaussian pulse. Maximum Pressure is 0.2 dB

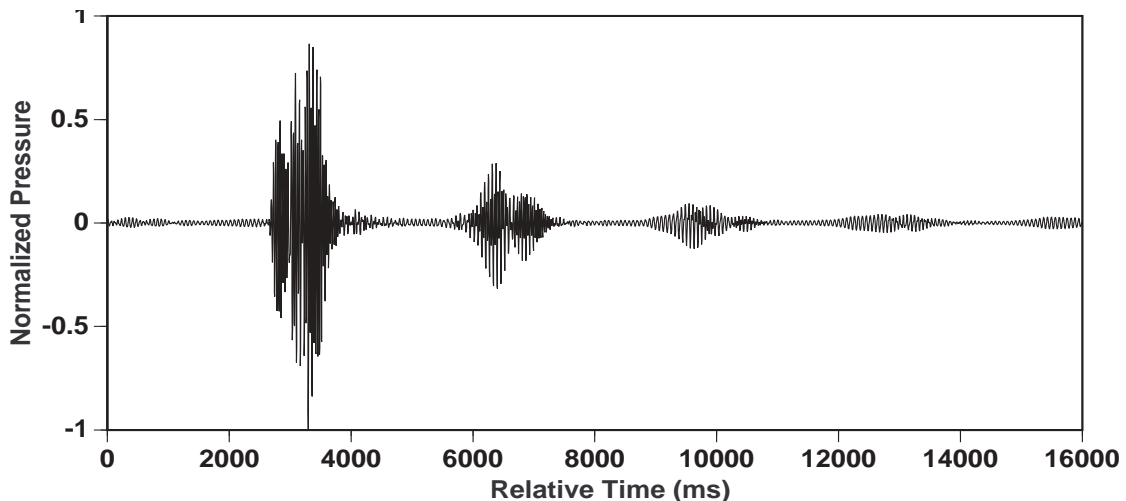
It is instructive to first consider the spectrum of the scattered field at the monostatic receiver for positive frequencies. The spectrum at negative frequencies is defined as the complex conjugate of the spectrum at the corresponding positive frequencies to assure a real signal. The amplitude of the spectrum of the scattered field at the monostatic receiver, plotted in dB versus frequency, is shown in Fig. 5. There is a strong component around 30 Hz because the corrugations have a period of 50 m, which is the wave length at 30 Hz. Another strong component appears near 15 Hz since the corrugations also have a period of 100 m. The component at 15 Hz is suppressed by the source spectrum, which is about 20 dB down at 15 Hz.



**Figure 5.** Spectrum of Scattered Field Shown as Received Level in dB versus Frequency

The finite extent of the scattering annulus should produce a predictable series of echoes (distinct from a uniform scattering layer extending to great range). The initial return should be at about 2.67 sec and is due to the path out to the scattering annulus at 2 km and back at a rate of about 1.5 km/sec.

The synthesized monostatic reverberation is shown in Fig. 6 where the normalized pressure is plotted versus time. The peak level, which occurs at about 3.33 sec is 44.3 dB down from a peak level of 0 dB at 1 m from the source. The main arrival starts at about 2.67 sec as expected. There are some arrivals from input pulses at earlier times indicating that the reverberation does not die out completely in the 16 sec interval. The width of the main arrival is a little more than 1 sec.



**Figure 6.** Time History of Test Case 5 at range 0.00 k. Maximum Peak is 44.3 dB

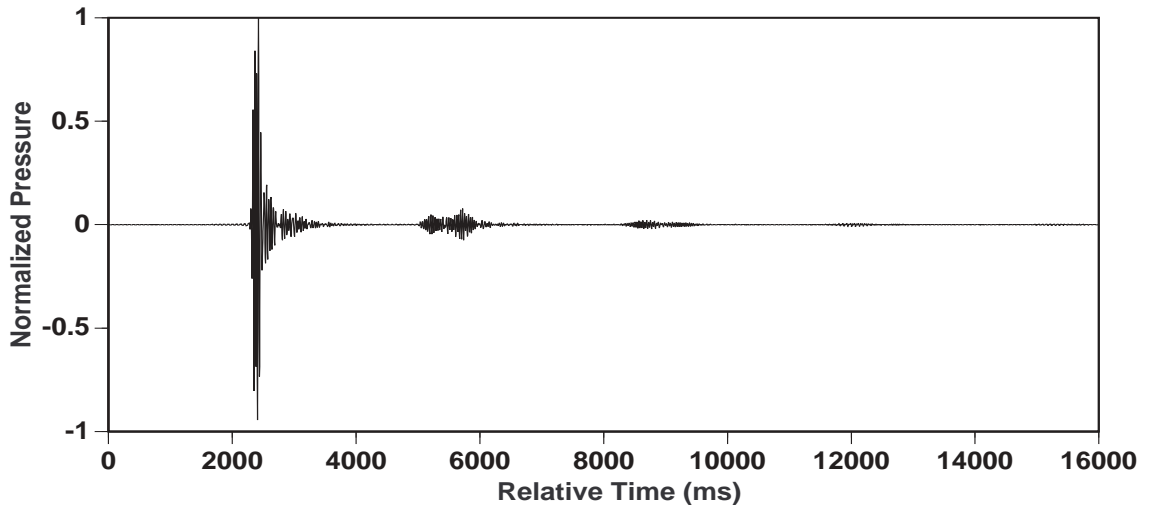
The things that contribute to the width of the echo are pulse length, modal dispersion and the extent of the scattering feature. The pulse length has already been described as 0.15 sec. Modal dispersion, at 30 Hz, based on the group velocities of the first and third modes (1.4903 and 1.3737 km/sec) for a two way path length of 4 km is 0.23 sec. The scatter extends 0.5 km in range so that scattering from the leading edge and trailing should be separated by 0.33 sec. Multiple scattering effectively increases the extent of the scattering feature. These factors combine to produce the echo of about 1 sec in duration.

The second event, in the reverberation series, should occur as a result of the first echo passing through the source position, encountering the scattering annulus and returning to the source position. The double round trip distance for this path is 8 km so the



second event should start around 5.33 sec. The second dominant arrival in Fig. 6 starts at about 5.8 sec which is consistent with group velocity the third mode at 30 Hz or a return from the midpoint of the scattering annulus.

A bistatic receiver at a depth of 45 m and range of 3.5 km outside the scattering annulus was also specified as part of Test Case 5. The bistatic receiver with the circular scattering annulus is of less physical interest, but the expected arrivals are easily identified. The received pulse, computed based on the total field, is shown in Fig. 7. The direct arrival, with a path length of 3.5 km, comes in at 2.33 sec and has a peak level of 61.2 dB down. A second return from the far side of the annulus, with a path length of 7.5 km, should arrive at about 5 sec and is evident in Fig. 7.



**Figure 7.** Time History of Test Case 5 at a range of 3.5 km.  
Maximum Peak is 61.2 dB

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### **References**

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